1. An Example of Bayesian Parameter Estimation

One of the easiest applications of Bayesian analysis is the estimation of the parameters of a statistical distribution. To see how it is done let us deal with a simple example: the exponential distribution

a) What is the exponential distribution of parameter λ?

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, x \geq 0 \\ 0, x < 0 \end{cases}$$

b) Write the general expression for likelihood of the sample vector $X=(x_1,\ldots,x_N)$

$$L(D) = \prod_{i=1}^{n} \lambda e^{-\lambda x_i} = \lambda^n e^{-\lambda \sum_{i=1}^{n} x}$$

c) It is customary to assign to scale parameters such as λ a prior distribution $P(\lambda) = \frac{1}{\lambda}$. Write the posterior of λ

$$P(\lambda \mid x) = \frac{P(x \mid \lambda) P(\lambda)}{P(x)}$$

d) Find the general expression of the maximum of the posterior

$$P(\lambda \mid x) = \lambda^{n-1} e^{-\lambda \sum_{i=1}^{n-1} x}$$

$$\ln P(\lambda \mid x) = (n-1) \ln \lambda - \lambda \sum_{i=1}^{n-1} x$$

$$\frac{d}{d\lambda} \ln P(\lambda \mid x) = (n-1)\frac{1}{\lambda} - \sum_{i=1}^{n-1} x,$$

at which its maximum occurs at $\frac{d}{d\lambda} \ln P(\lambda \mid x) = 0$

$$\frac{1}{\lambda} = \frac{1}{(n-1)} \sum_{i=1}^{n-1} x_i$$

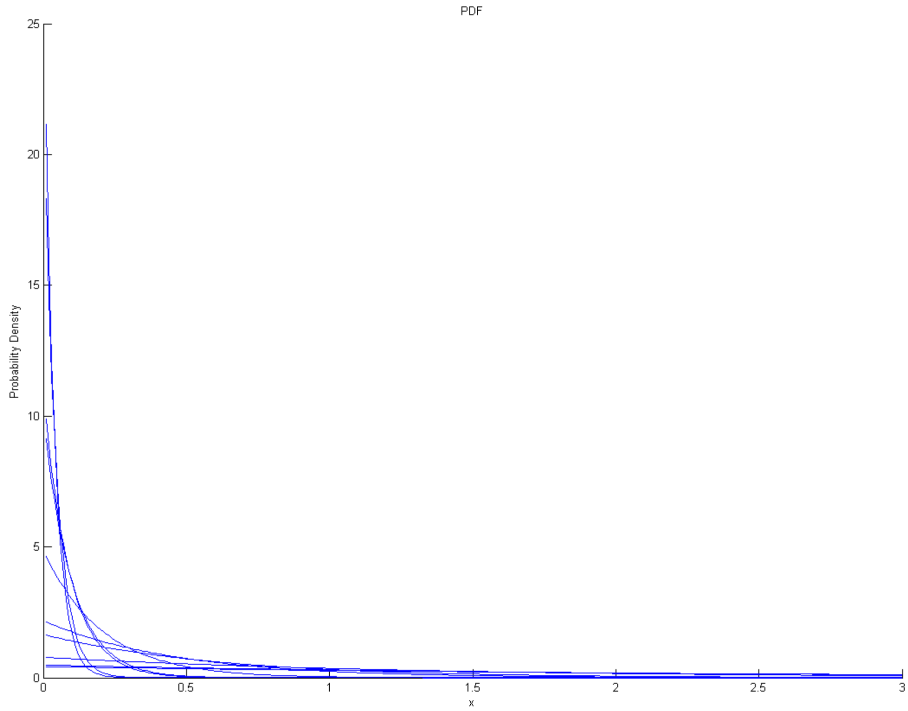
e) **Bonus Question:** P(λ)=1/ λ is not normalised… What can be done about it?

To normalise, $P(\lambda) = \frac{1/\lambda}{\int_0^\infty \lambda d\lambda}$

f) Draw 10 samples $x_1,\ldots,x_{10}$ from the exponential distribution λ =1

0.2049	0.0989	2.0637	0.0906	0.4583
2.3275	1.2783	0.6035	0.0434	0.0357

g) For these 10 samples plot the posterior of λ



h) Calculate its maximum, its expected value and its standard deviation