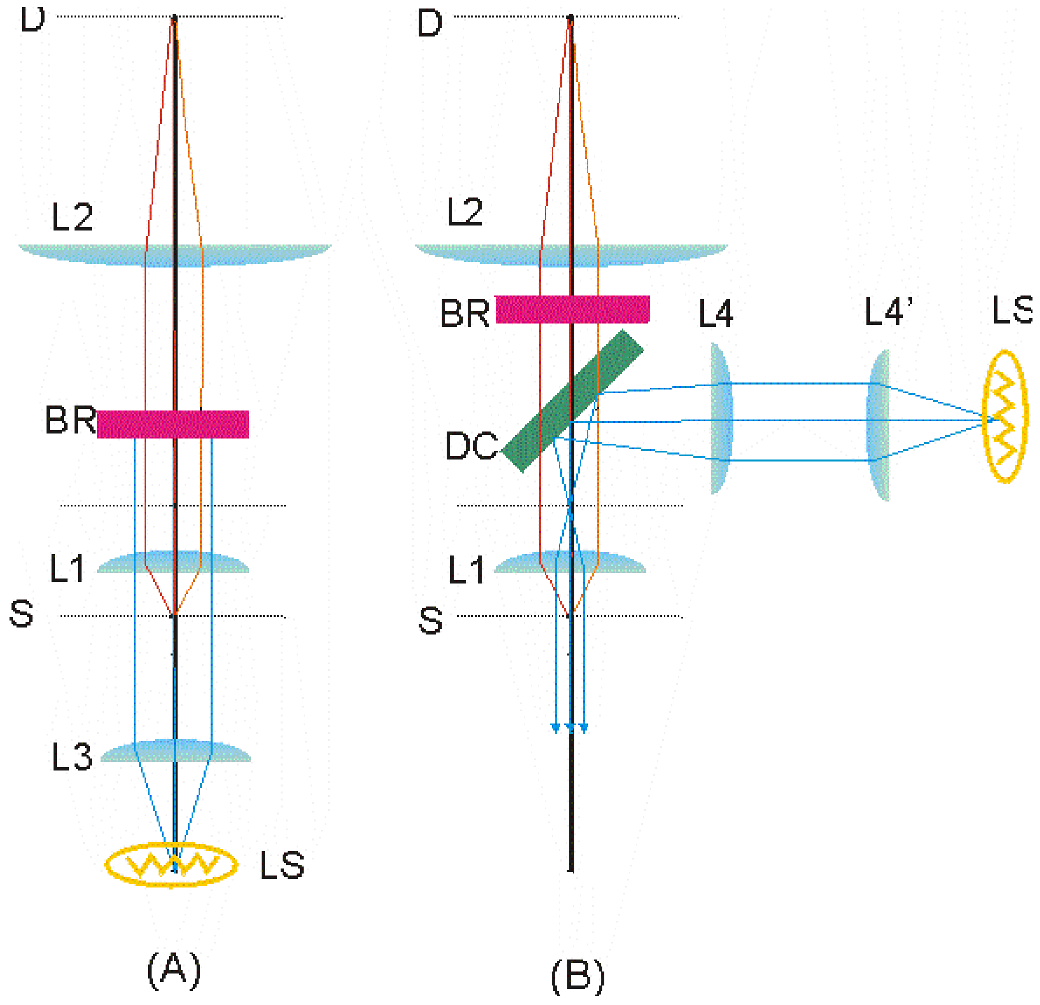
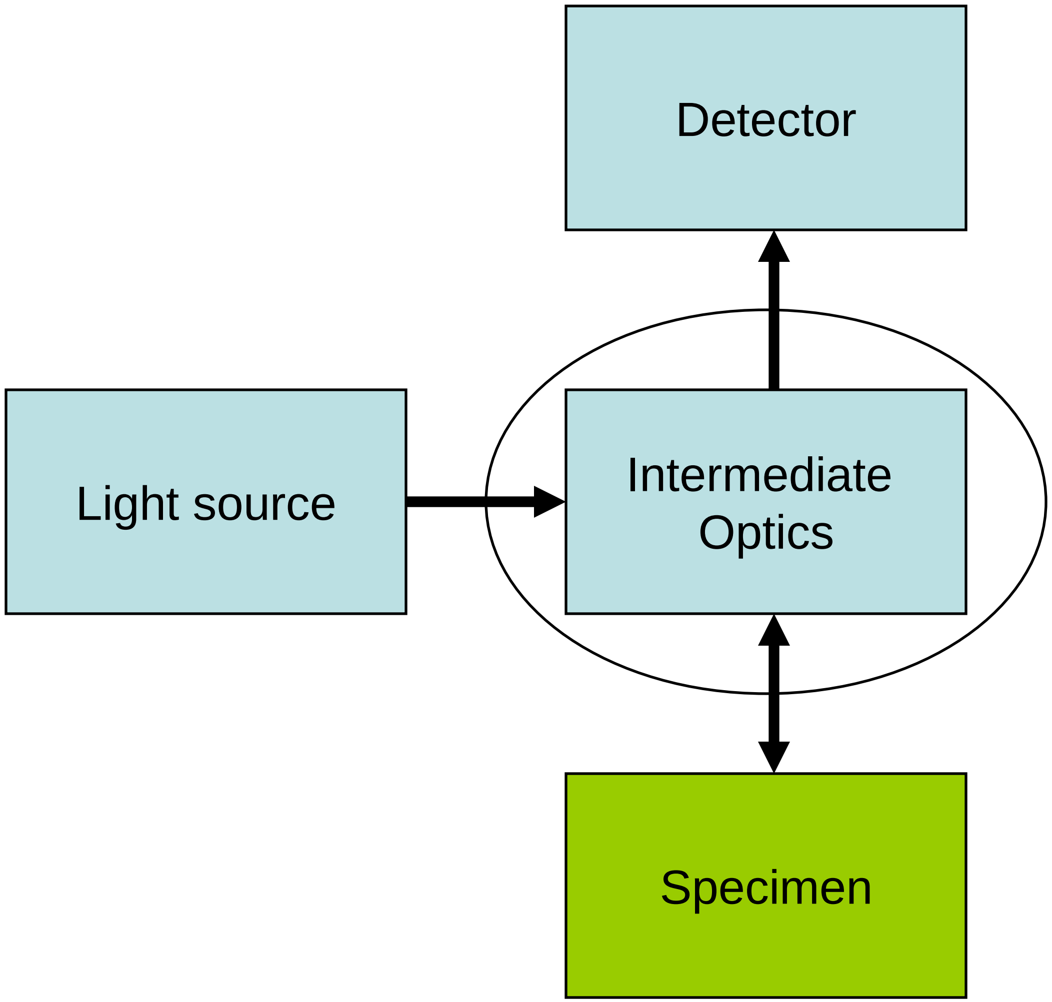
Optics & Microscopy I

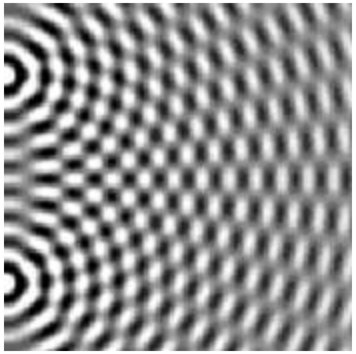


A typical biomedical optics experiment

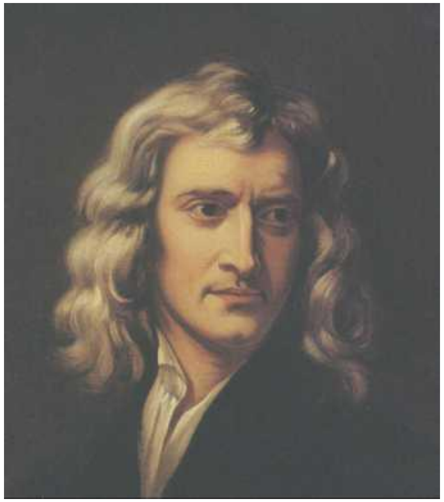


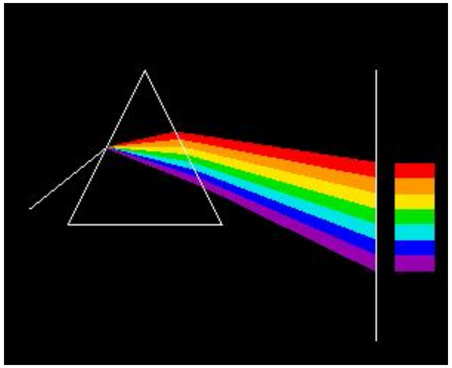
Wave and Particlel Nature of Light

Wave Nature of Light -- Huygen



Particle Nature of Light -- Newton





Physical Optics – Wave nature of light

Maxwell and His Equations

$$\nabla \cdot \vec{E} = 4\pi\rho$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} = 0$$

$$\nabla \times \vec{B} - \frac{1}{c}\frac{\partial \vec{E}}{\partial t} = \frac{4\pi}{c}\vec{J}$$

Wave Equations

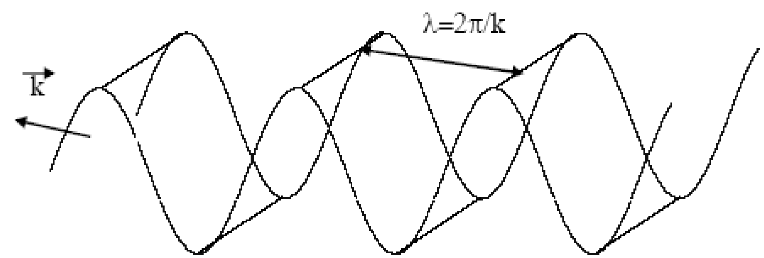
$$\nabla^2 \vec{E} - \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

$$\nabla^2 \vec{B} - \frac{1}{c^2}\frac{\partial^2 \vec{B}}{\partial t^2} = 0$$

Plane Wave Solution

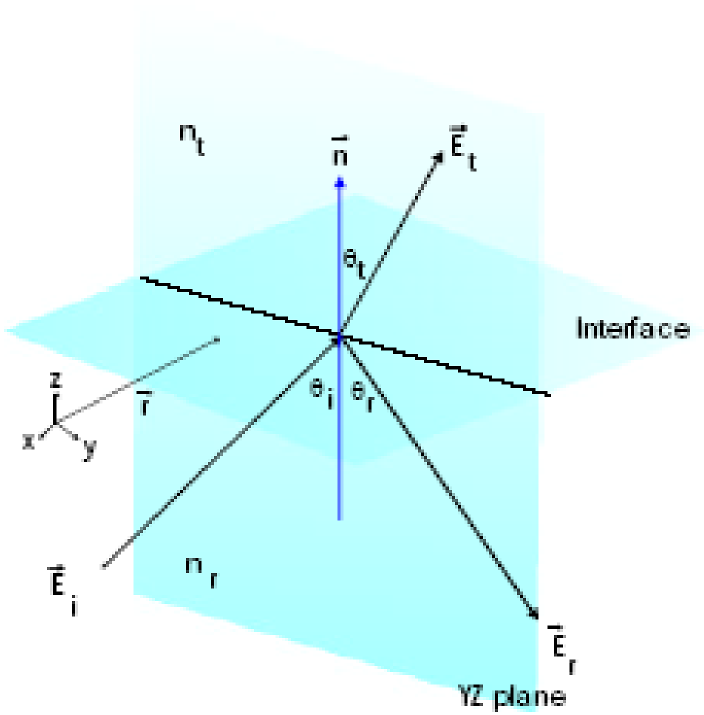
$$\vec{E}(x,t) = \vec{E}_0 \cos(kx - \omega t)$$

$$k = \frac{2\pi}{\lambda} \quad \omega = 2\pi f \quad ck = \omega$$



Plane wave propagates like a “ray” of light

Reflection and Refraction of Light at Boundary



Reflection

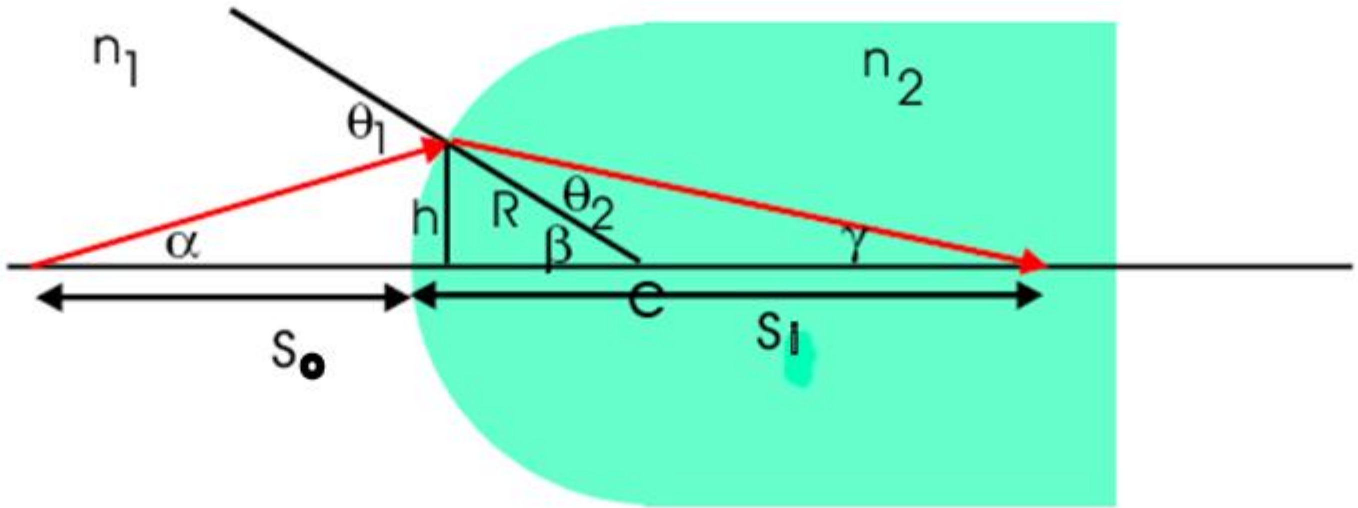
$$\sin\theta_i = \sin\theta_r$$

Refraction (Snell's Law)

$$n_i \sin\theta_i = n_t \sin\theta_t$$

Refraction at a spherical surface

Let's look at one of the simplest case of how light is transmitted through a spherical dielectric (glass)-air interface



We can trace a ray originating at a distance S_1 from the interface. How far from the interface (S_2) will the ray intersect the axis of the spherical interface?

This question can be settled by Snell's law and ray tracing:

From Snell's Law: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

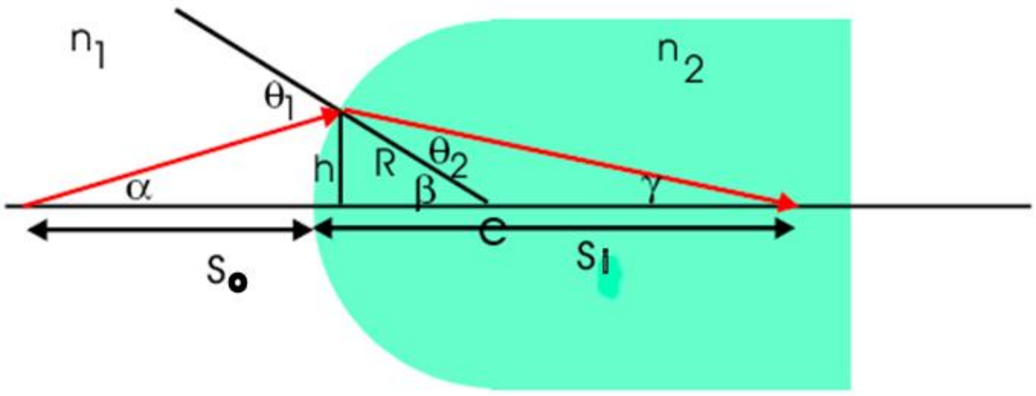
Also from geometry: $\theta_1 = \alpha + \beta$ and $\theta_2 = \beta - \gamma$.

The solution is complicated unless all the angles are small. We will first consider this case. In this case, the sine of the sum of two small angles is the sum of the sines:

$$\sin\theta_1 = \sin(\frac{h}{S_0}) + \sin(\frac{h}{R}) = \frac{h}{S_0} + \frac{h}{R}$$

$$\sin\theta_2 = \sin(\frac{h}{R}) - \sin(\frac{h}{S_i}) = \frac{h}{R} - \frac{h}{S_i}$$

From Snell's law:



$$\frac{n_1}{S_0} + \frac{n_1}{R} = \frac{n_2}{R} - \frac{n_2}{S_i}$$

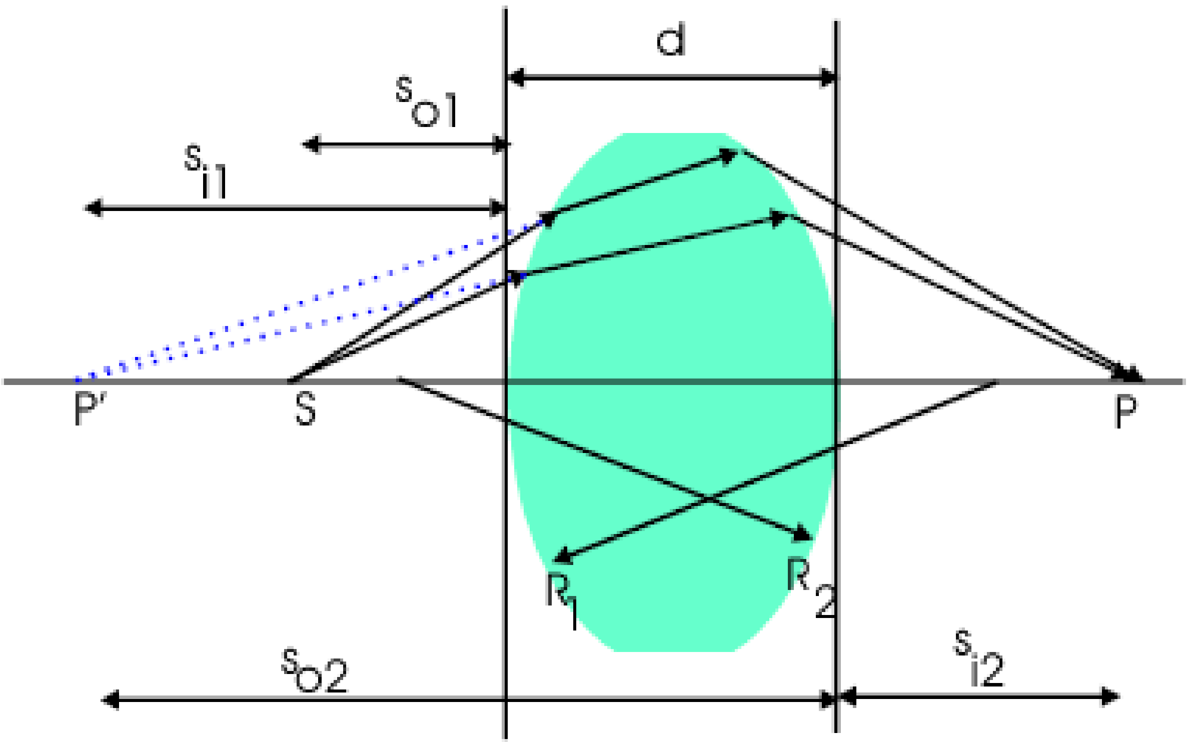
This simplies to:

$$\frac{n_1}{S_0} + \frac{n_2}{S_i} = \frac{n_2 - n_1}{R} \qquad (1)$$

The assumption of small angle is called the paraxial approximation.

Ray tracing through a lens

A combination of two dielectric interfaces:

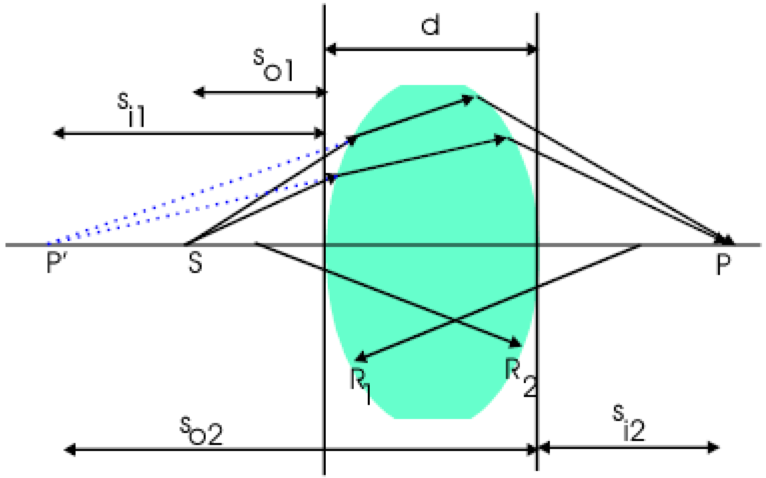


For the first interface, we form a virtual image at P'.

From (1), we have:

$$\frac{n}{s_{o1}} + \frac{n'}{s_{i1}} = \frac{(n - n')}{R_1} \qquad (2)$$

Note that S_{i1} is negative as it is virtual. For the 2nd interface, the incoming rays will appear to originate from P'. Therefore the object distance, So2, is



$$s_{o2} = d - s_{i1} = d + |s_{i1}|$$

and from (1) again:

$$\frac{n'}{d + s_{i1}} + \frac{n}{s_{i2}} = \frac{(n' - n)}{R_2} \qquad (3)$$

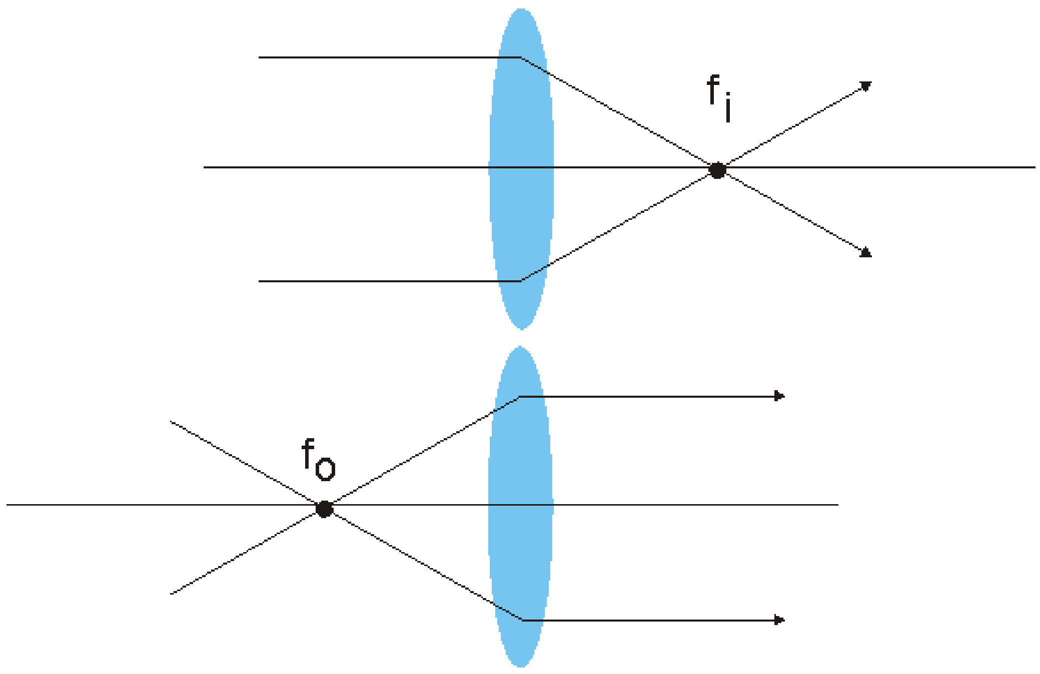
Adding (2) & (3), we got:

$$\frac{n}{s_{o1}} + \frac{n}{s_{i2}} = (n' - n)\left(\frac{1}{R_1} - \frac{1}{R2}\right) + \frac{n'd}{(s_{i1} - d)s_{i1}} \qquad (4)$$

For the case of thin lens, the last term can be neglected. Further, if the lens is place in air, n=1, we get the familiar Lensmaker's equation.

$$\frac{1}{S_{o1}} + \frac{1}{S_{i2}} = (n' - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Definition of focal points for a thin lens



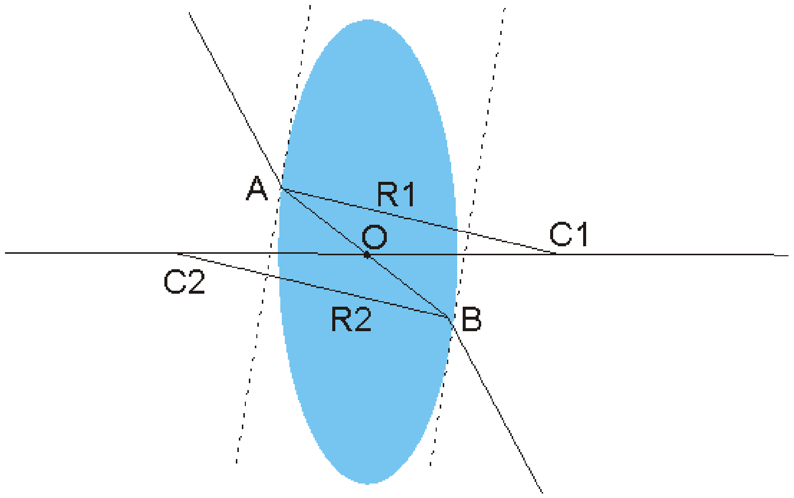
$$\lim_{s_o \to \infty} s_i = (n' - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = f_i$$

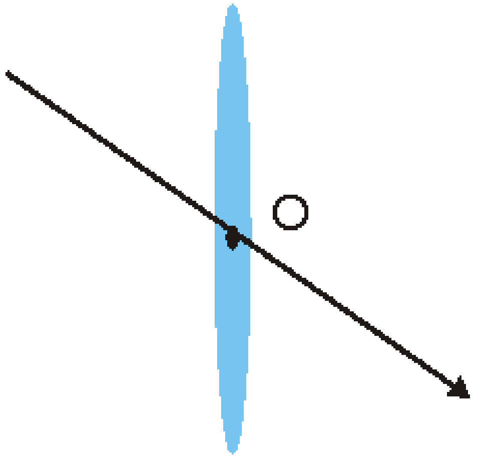
$$\lim_{s_i \to \infty} s_0 = (n' - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = f_o$$

$$f_o = f_i = f$$

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

Defining Optical Center of a Lens





For any pair of parallel planes tangent to the lens surface, we can find two contact points A, B

The optical center O is the intersection of line AB with the optical axis

Note that the triangles AOC1 and BOC2 are similar. Therefore, $\overline{R1} \cdot \overline{OC1} = \overline{R2} \cdot \overline{OC2}$

Since R1 and R2 are constants, the position of O is constant independent of the choice of A,B

From Snell's Law, the direction of the ray entering the lens is the same as that leaving the lens

If the lens is thin, the parallel deviation is small and we can draw the rays going through
the optical center as straight lines

Rules for Ray Tracing

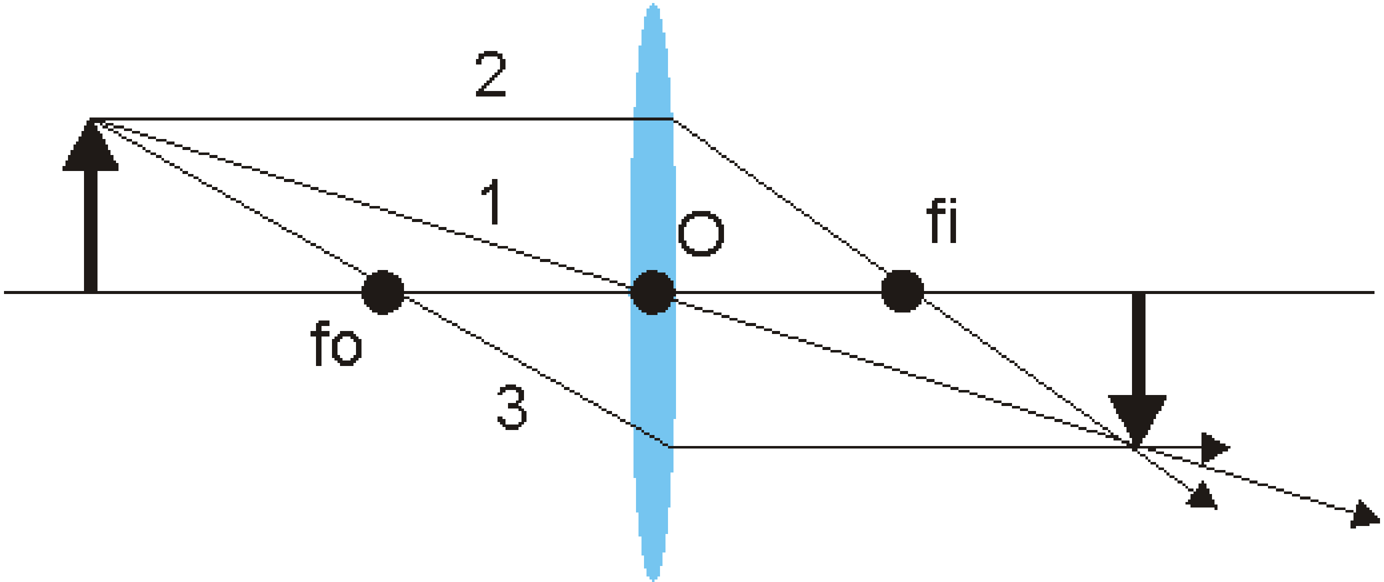
(1) Draw the object to be imaged, the optical axis, the lenses (with their front & back focal points) and pick a point on the object.

(2) Trace the first ray as a straight line going through the optical center of the lens.

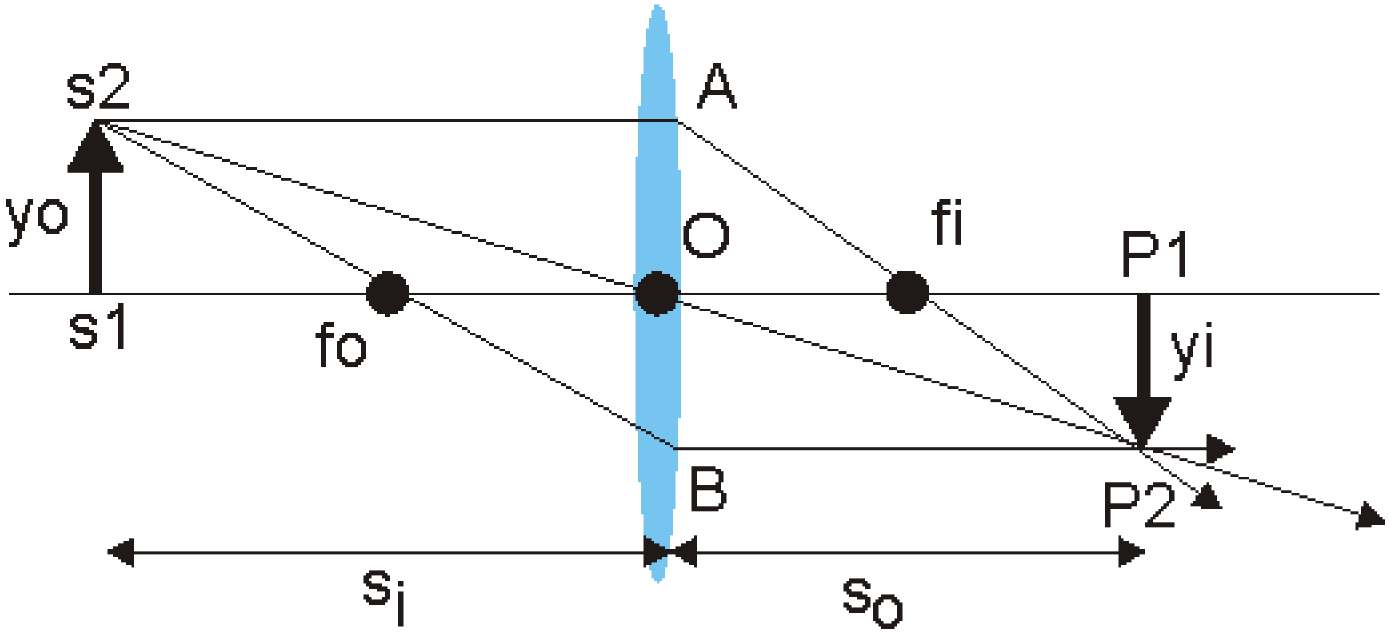
(3) Trace the second ray going parallel to the optical axis until it intersect the lens, then it goes through its back focal point.

(4) Trace the third ray going from the object to the front focal point until it intersects the lens. After the lens, its goes parallel to the optical axis.

(5) The intersection point of the 3 rays is the image of the point on the object



Ray tracing & Lens Making's Formula

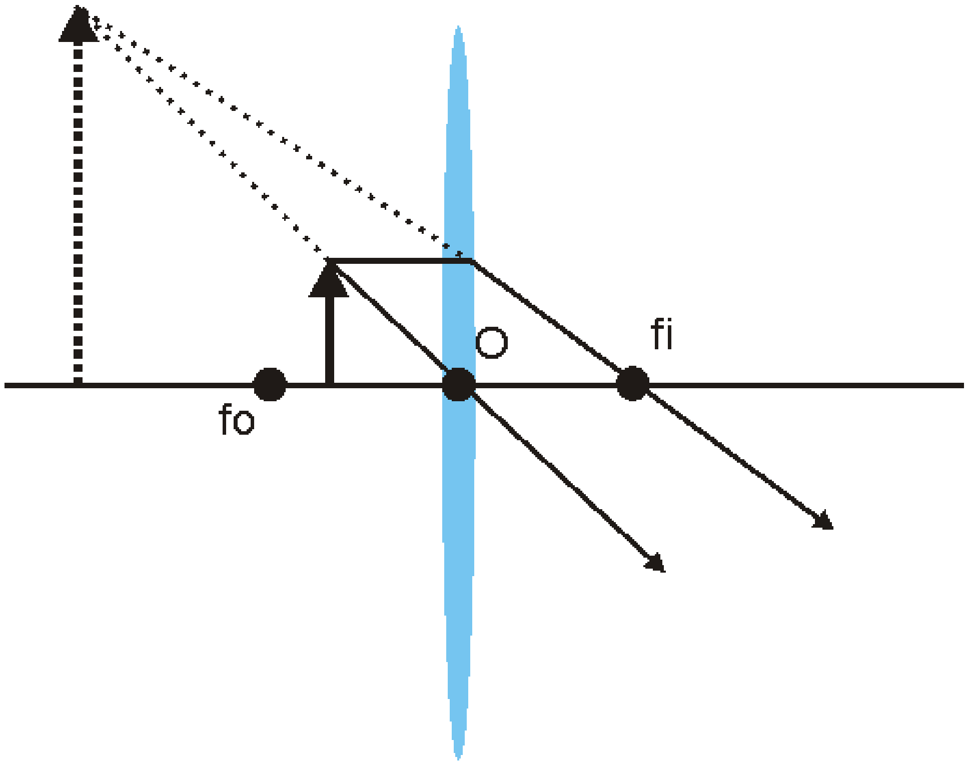


$$AOf_i \sim P_1P_2f_i \rightarrow \frac{y_o}{y_i} = \frac{f}{s_i - f}$$

$$s_1s_2O \sim P_1P_2O \rightarrow \frac{y_o}{y_i} = \frac{s_o}{s_i}$$

$$\frac{f}{s_i - f} = \frac{s_o}{s_i} \rightarrow \frac{1}{s_i} + \frac{1}{s_o} = \frac{1}{f}$$

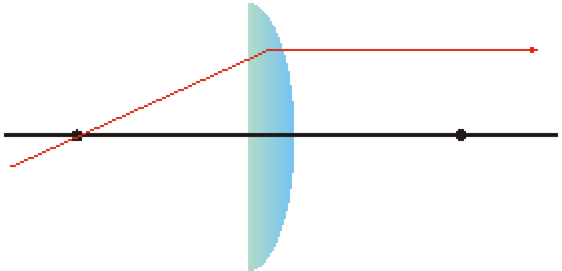
Another example in ray tracing



The image is “virtural”.

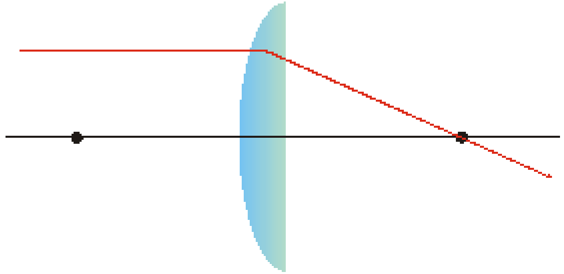
Simpler Ray Tracing Rules I

Rays pass through the focal point becomes parallel to the optical axis.
Rays parallel to the optical axis are deflected through the focal point.



(1)

Rays originated from the focal point emerge parallel to the optical axis

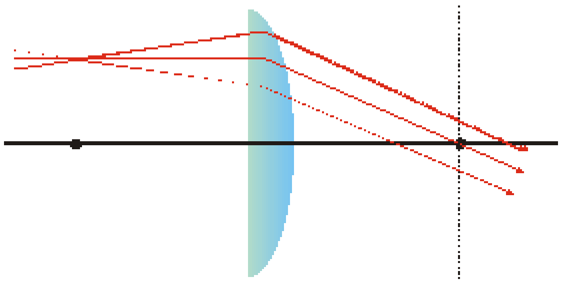


(2)

Rays parallel to the optical axis converges to the focal point

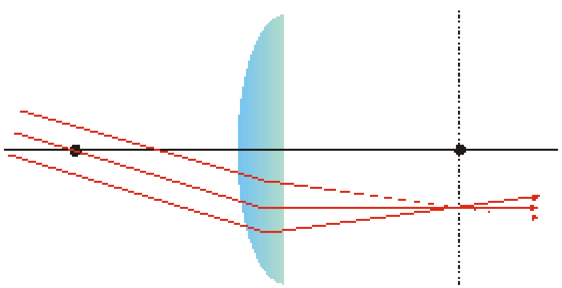
Simpler Ray Tracing Rules

Rays originate from the focal plane becomes collimated.
Collimated rays converges at the focal plane.



(3)

Rays originated from the plane emerge collimated



(4)

Collimated rays emerge focus at The focal plane