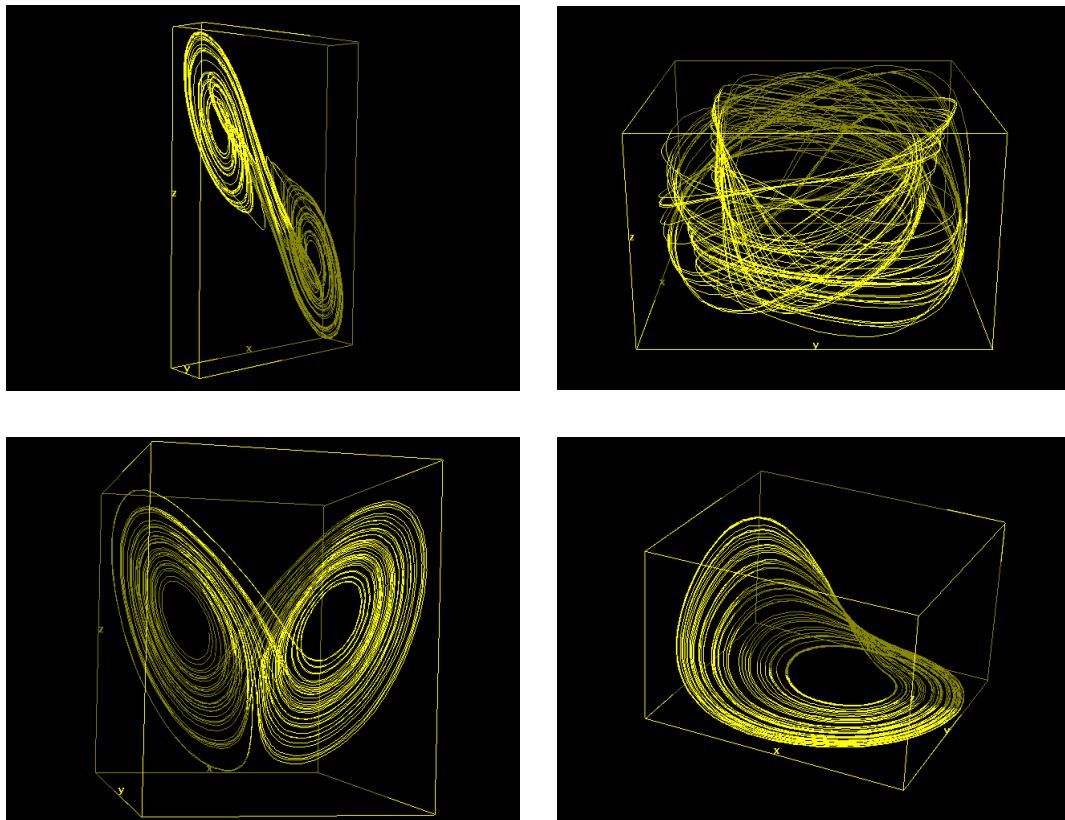


Template for the Analysis of a Dynamic System



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Part 1: Model Description

1.1) Background about the Model

As an introduction to your study describe with as many details as possible:

- the History of the Model
- the Tradition Areas of Application of the Model
- the Relevance of the Model for your Project?

Bibliographical references are always welcome. Likewise all famous results about the model may be quoted.

1.2) Model description

- Write down the set of ODEs for your model
- Give a Physical Interpretation of the Model
 - What are the Basic Assumptions of the Model?
 - Describe in a table the signification of each term and parameter of the model
 - Use simple terms
- Discuss Strengths and Weaknesses of the Model
 - Insist on physical interpretation
 - Look into relevance of model for small numbers

1.3) Make the System Simpler

In the case of complex systems with many different parameters it is advised to simplify the dynamic system and make it simpler. This is achieved by using a well-chosen reversible, continuous (possibly non-linear) mapping of the variables (time and vector X) into a different set of variables (α and vector U).

Most of the time the mapping is linear that is every variable (including the time) is multiplied by a well-chosen constant – it is common to refer to the new system as dimension-less.

For instance turn the following (complex) system (8 parameters)

$$(\text{Prey} - \text{Predator}) \quad \begin{cases} \frac{dX}{dt} = \frac{aX}{a_0 + X} - b \frac{XY}{b_0 + X} - eX \\ \frac{dY}{dt} = c \frac{XY}{c_0 + XY} - dY \end{cases}$$

into the 'nicer' (only 5 parameters)

$$(\text{Prey} - \text{Predator 2}) \quad \begin{cases} \frac{dU}{d\alpha} = \frac{AU}{1_0 + U} - B \frac{UV}{B_0 + U} - EU \\ \frac{dV}{d\alpha} = C \frac{UV}{1 + UV} - DV \end{cases}$$

Part 2: Theoretical Model Analysis

The dynamic analysis is akin to the study of the remarkable points of a function. This is done in two steps detailed as in the WIKI document 1. The study of the behaviour at infinity is the complement and is detailed in WIKI document 2.

2.1) Studying the Existence of Steady Points

- Location, Number...
- Conditions for Existence etc...

2.2) Studying the Stability of the Steady Points

- Analysis of Sign of Trace and Determinant of Jacobian in 2D
- Finding Eigenvalues of Jacobian
- Hough-Horowitz...

2.3) Behaviour at Infinity

- Poincare – Bendixson in 2D
- Are Trajectories bounded? etc....

2.4) Sketch the Vector Field (VF)

- Keep it as Theoretical as Possible (NO Simulations if Possible)
- Make Summary of all your Findings
- In particular :
 - Describe all the Possible Cases
 - Relate these Cases to the Model Parameters

Part 3: Computer Simulations

3.1) Qualitative Complements on the Vector Field

- Run Simulations to Complement your Analysis of the VF
In particular Focus on
 - Shape of Trajectories
 - Influence of initial conditions
 - Influence of model parameters
- Make predictions and suggestions regarding the sensitivity analysis

3.2.) Quantitative Analysis of the Vector Field

The most remarkable characteristics of the Vector Field should be identified by now. They are of two different kinds. The first kind contains the components that depend on the model parameters only (for instance a limit cycle or the basin of attraction of a stable steady point). The second kind of properties depend on initial conditions as well – without oversimplifying we could say they sum up the way the rest of the Vector Field arranges itself with regards to the objects of the first kind – for instance speed of convergence of trajectory toward a limit cycle or a steady point.

There is of course no global template for the quantitative analysis of a Vector Field, only principles based on common sense and experience.

for instance here are some of the more useful characteristics for the Study of a Limit Cycle :

- Geometric Measures (in X,Y plane)
 - position of centre
 - eccentricity, main curvature
 - distance to centre (minimum, maximum, average, standard deviation...)
 - length, area ...
- 1D Signal Analysis of X(t), Y(t)..
 - period / Fourier Analysis / Wavelet Analysis...
 - Average, extrema, amplitude, moments about the mean..