

Journal Week 4

$$\begin{aligned}\frac{dx}{dt} &= -qx + V \frac{y}{k+y} \frac{z}{m+z} x \\ \frac{dy}{dt} &= qu - qy - \frac{1}{\epsilon} V \frac{y}{k+y} \frac{z}{m+z} x \\ \frac{dz}{dt} &= qv - qz - \frac{1}{\delta} V \frac{y}{k+y} \frac{z}{m+z} x\end{aligned}$$

1) $\frac{dx}{dt} = -qx + V \frac{y}{k+y} \frac{z}{m+z} x = 0$
 $x=0$

$$-q + V \frac{y}{k+y} \frac{z}{m+z} = 0$$

$$q = V \max \left\{ \frac{y}{k+y}, \frac{z}{m+z} \right\}$$

$$qu - qy = \frac{1}{\epsilon} qx$$

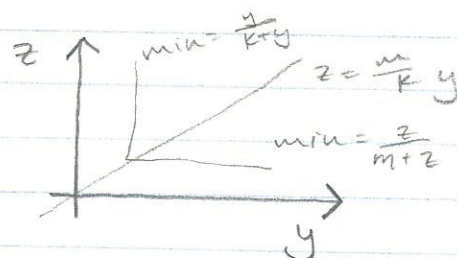
$$qv - qz = \frac{1}{\delta} qx$$

$$\min \left\{ \frac{y}{k+y}, \frac{z}{m+z} \right\} = \frac{a}{V \max}$$

from $\frac{dy}{dt} = 0$
 $\begin{cases} x = \epsilon(u-y) \\ x = \delta(v-z) \end{cases}$

$$\epsilon(u-y) = \delta(v-z)$$

$$\boxed{z = \frac{\epsilon}{\delta} y + v - \frac{\epsilon}{\delta} u}$$



2) $\frac{dy}{dt} = qu - qy - \frac{1}{\epsilon} V \frac{y}{k+y} \frac{z}{m+z} x$
 $\frac{dz}{dt} = qv - qz - \frac{1}{\delta} V \frac{y}{k+y} \frac{z}{m+z} x$

Same process as above in class

$$z = \frac{\epsilon}{\delta} y + v - \frac{\epsilon}{\delta} u = \boxed{v - \frac{\epsilon}{\delta} (u-y)}$$

3) $z = \frac{\epsilon}{\delta} y$ (equilibrium)

$$z = v - \frac{\epsilon}{\delta} (u-y)$$

For y: $\frac{\epsilon}{\delta} y = v - \frac{\epsilon}{\delta} (u-y)$

$$\frac{\epsilon}{\delta} y = v - \frac{\epsilon}{\delta} u + \frac{\epsilon}{\delta} y$$

$$y \left(\frac{\epsilon}{\delta} + \frac{\epsilon}{\delta} \right) = v - \frac{\epsilon}{\delta} u$$

$$\boxed{y = \frac{v - \frac{\epsilon}{\delta} u}{\frac{\epsilon}{\delta} + \frac{\epsilon}{\delta}}}$$

$$y = \frac{kvd - \epsilon ku}{m\delta + \epsilon k} = \frac{k(v\delta - \epsilon u)}{m\delta + \epsilon k}$$

3) cont.

$$z = v - \frac{\epsilon}{f}(v-y)$$

$$z = v - \frac{\epsilon}{f}v - \frac{\epsilon}{f}y$$

$$\frac{z - v + \frac{\epsilon}{f}v}{-\frac{\epsilon}{f}}$$

$$\frac{-z + v - \frac{\epsilon}{f}v}{\frac{\epsilon}{f}} = y$$

$$z = \frac{m}{k}y$$

$$y = z \frac{k}{m}$$

$$\frac{-z + v - \frac{\epsilon}{f}v}{\frac{\epsilon}{f}} = z \frac{k}{m}$$

$$-z + v - \frac{\epsilon}{f}v = z \frac{k}{m} \cdot \frac{\epsilon}{f}$$

$$v - \frac{\epsilon}{f}v = z \frac{k}{m} \frac{\epsilon}{f} + z$$

$$v - \frac{\epsilon}{f}v = z \left(\frac{k}{m} \frac{\epsilon}{f} + 1 \right)$$

$$z = \frac{v - \frac{\epsilon}{f}v}{\frac{k}{m} \frac{\epsilon}{f} + 1}$$

4) $y = \frac{k(vf - \epsilon v)}{mf + \epsilon k}$

$$y = \mu - \frac{f}{\epsilon}(v - z)$$

$$\frac{k(vf - \epsilon v)}{mf + \epsilon k} = \mu - \frac{f}{\epsilon}(v - z)$$

$$\frac{k(vf - \epsilon v)}{mf + \epsilon k} = \mu - \frac{f}{\epsilon}v + \frac{f}{\epsilon}z$$

$$\frac{k(vf - \epsilon v)}{mf + \epsilon k} - \mu + \frac{f}{\epsilon}v = \frac{f}{\epsilon}z$$

$$x = fv - fz$$

$$z = \frac{\epsilon}{f} \left(\frac{k(vf - \epsilon v)}{mf + \epsilon k} + \mu - f \cdot v \right)$$

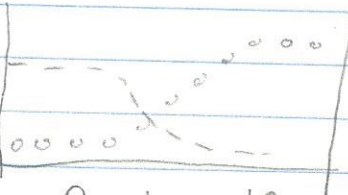
$$x = fv - f \left(\frac{\epsilon}{f} \left(\frac{k(vf - \epsilon v)}{mf + \epsilon k} \right) + \mu - fv \right)$$

$$x = \cancel{fv} - \frac{\epsilon k(vf - \epsilon v)}{mf + \epsilon k} + \mu - \cancel{fv}$$

$$x = \frac{-\epsilon k(vf - \epsilon v)}{mf + \epsilon k}$$

5)

biomass
concentration



feed rate

--- Nutrient
concentration