Addendum for Problem Set #5

In lecture, we modeled a mass on a spring as

$$\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = F_d / \mathrm{m}$$

where

$$\gamma = {b}/{2m} \quad \text{and} \quad \omega_o^2 = {k}/{m}$$

You will find that your solution for Part (a) will be either oscillatory with decaying amplitude or purely decaying depending on whether γ is less than or greater than ω_o. While this problem can be worked for either (or both) cases, I recommend that you assume $\gamma < \omega_o$ which will give a decaying oscillatory solution. This is equivalent to defining the problem as an underdamped oscillator (as opposed to overdamped) and can be used to model the microcantilever for your lab module.

One thing I never stated explicitly in lecture is that since the Fourier transform of the delta function $\delta(t)$ is one, the units for $\delta(t)$ are $\sec^{-1}$. As a result, the impulse response for a system h(t) will have units of sec. While expressing h(t) is not an absolute convention, it will make life easier to stick to it.

I realized Part (b), while correct as written, does not follow this convention and this can be a source for confusion. It would be better for this part to be written as:

(b) The initial conditions can be realized experimentally by accelerating the end of the cantilever with a very short impulse, $g(t) = v_o \cdot \delta(t)$:

$$\ddot{f} + 2\gamma\dot{f} + \omega_0^2 f = v_o \cdot \delta(t)$$

where $f = x(t)$ is defined as the output of the system. We can rewrite this equation as

$$\ddot{h} + 2\gamma\dot{h} + \omega_0^2 h = \delta(t)$$

where h(t) is the impulse response. The output from the impulse acceleration is now given by $f(t) = v_o \cdot h(t)$.

Other notes

<u>Parts d and g</u>: It is not necessary to know the initial velocity, v_o.

<u>Parts d and e</u>: Make plots for various values of γ. In class, we discussed how the quality factor Q relates to the frequency response. Keep in mind that the quality factor is defined as, $Q = \omega_o / 2\gamma$

<u>Part e</u>: The sampling window will depend on γ and should be somewhere in the range of 1-10 ms.