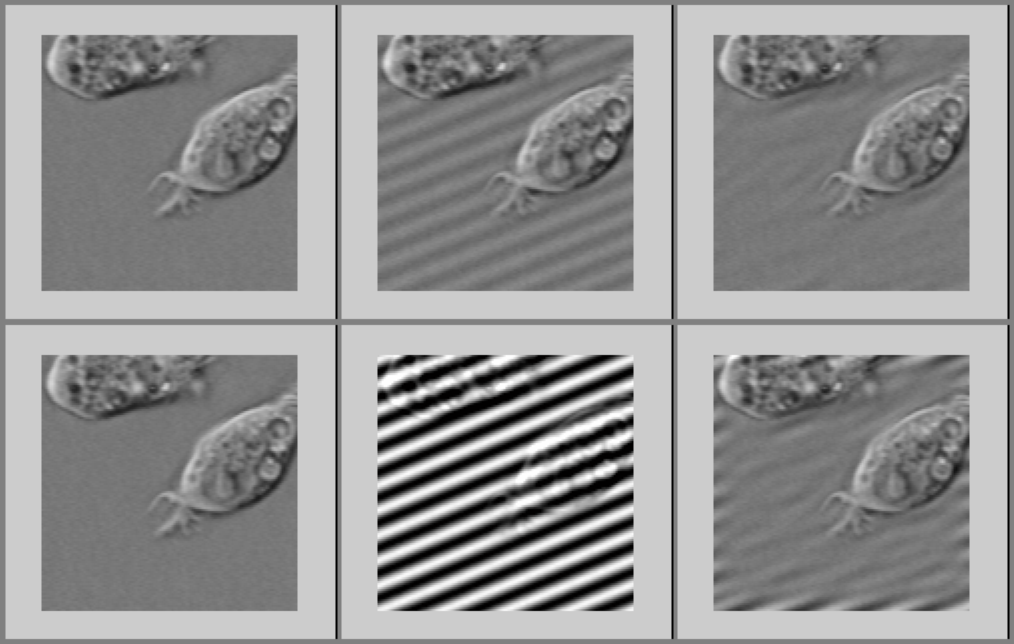
Image Processing and Analysis II



Materials extracted from Gonzalez & Wood
and Castleman

Image Preprocessing – Morphological Operations I

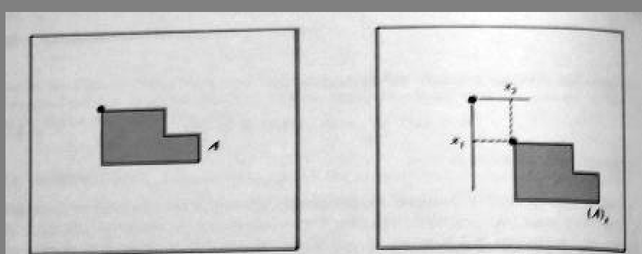
Let A and B be a set of points in the image.

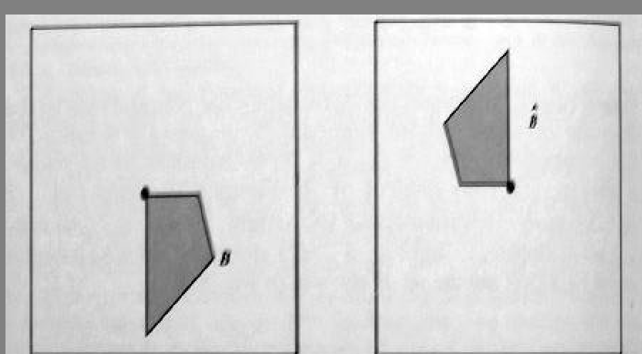
Translation of A by a vector x: $(A)_x = \{c \mid c = a + x, \quad for\ a \in A\}$

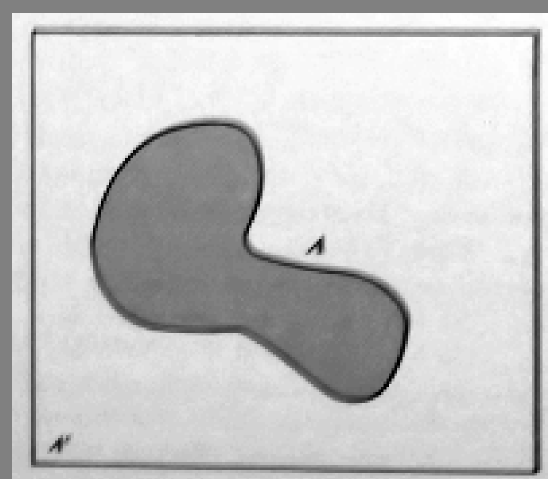
Reflection of B: $\hat{B} = \{x \mid x = -b, \quad for\ b \in B\}$

Complement of A: $A^c = \{x \mid x \notin A\}$

Difference of A & B: $A - B = \{x \mid x \in A\ and\ x \notin B\}$







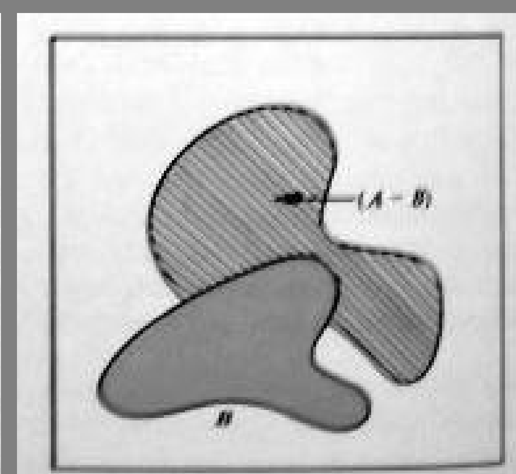
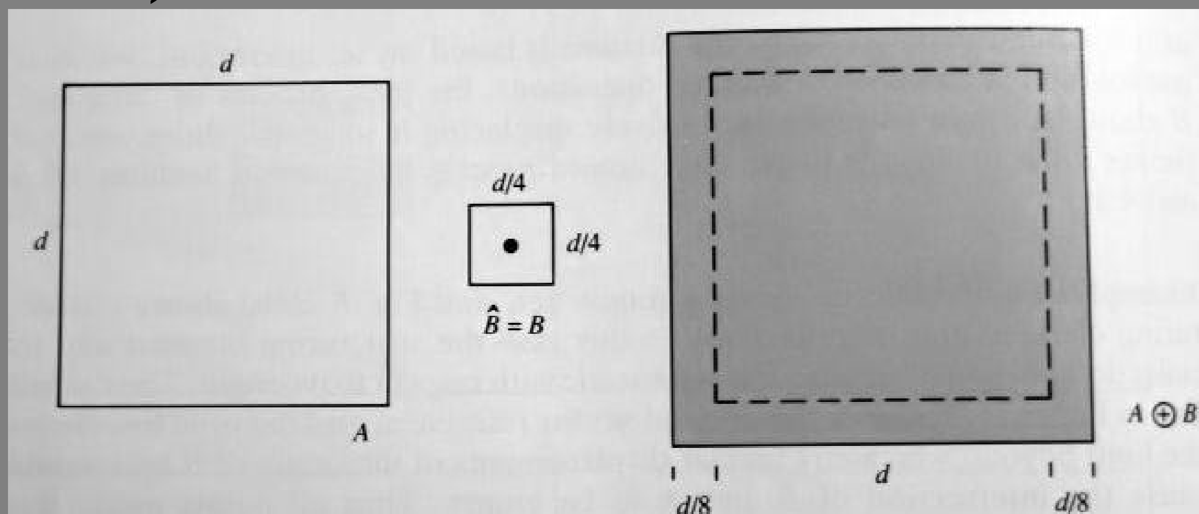


Image Preprocessing – Morphological Operations II

Diluation of A by B: $A \oplus B = \{x \mid [(\hat{B})_x \cap A] \neq f\}$

Erosion of A by B: $A \Theta B = \{x \mid (B)_x \subseteq A\}$

Note $(A \Theta B)^c = A^c \oplus \hat{B}$



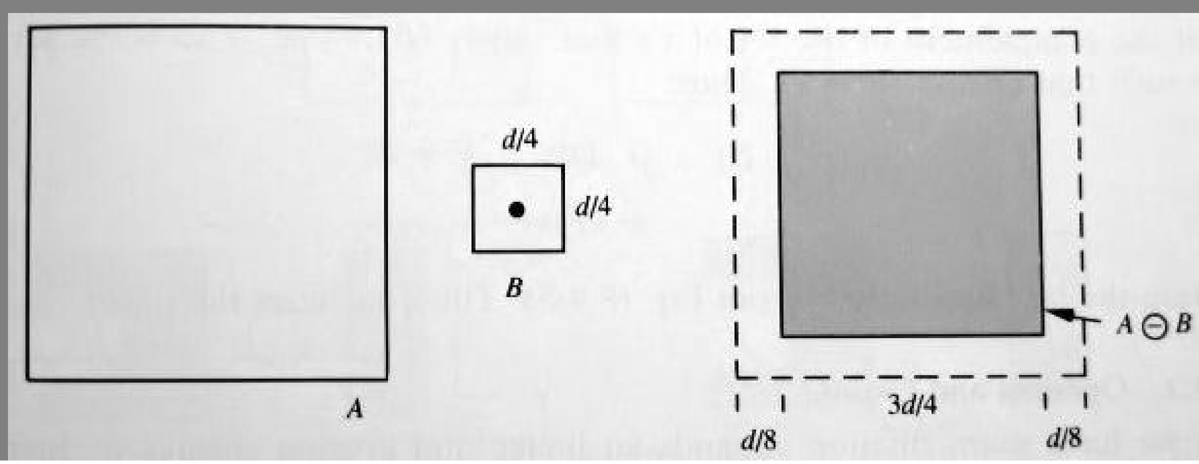
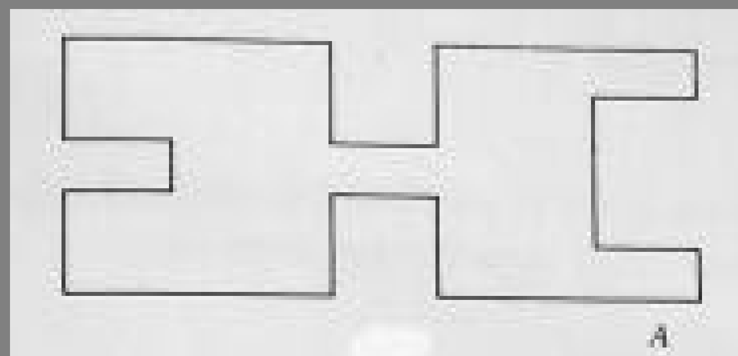


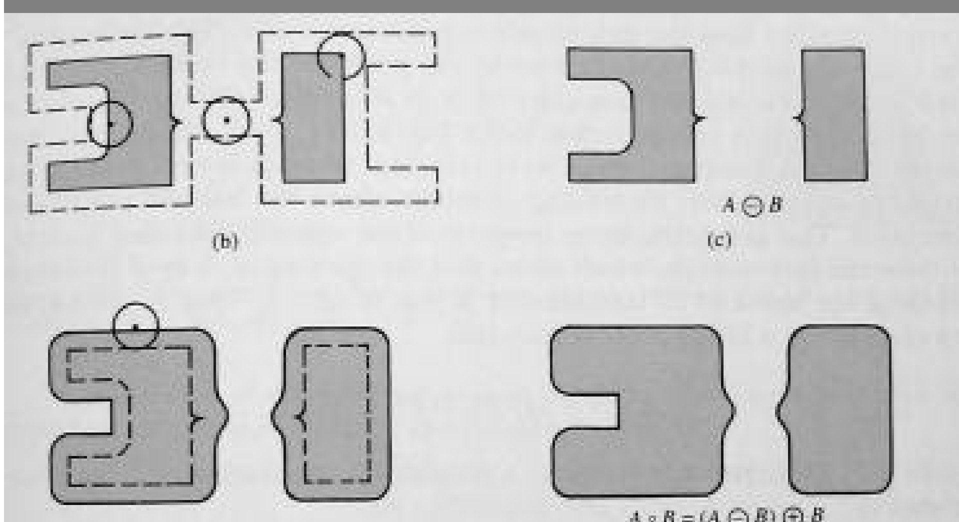
Image Preprocessing – Morphological Operations III

Opening of A by B: $A \circ B = (A \Theta B) \oplus B$

Closing of A by B: $A \bullet B = (A \oplus B) \Theta B$

Note $(A \circ B) \circ B = A \circ B$ and $(A \bullet B) \bullet B = A \bullet B$





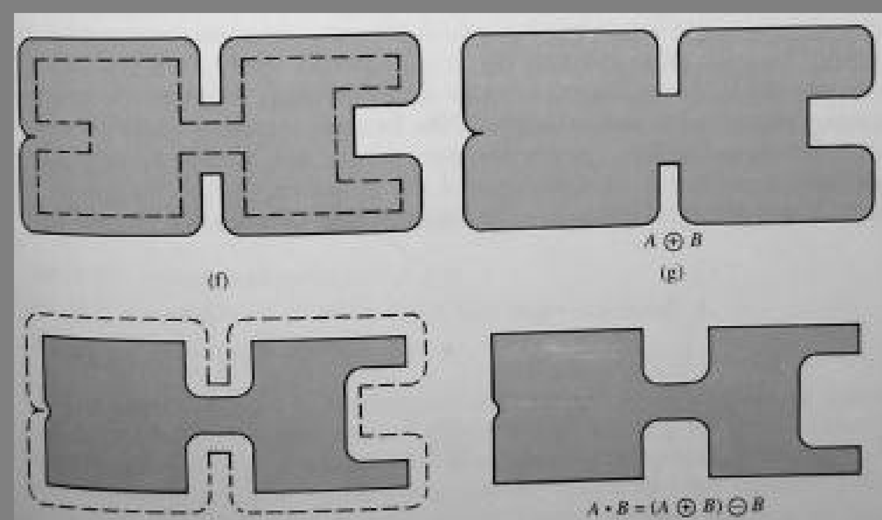
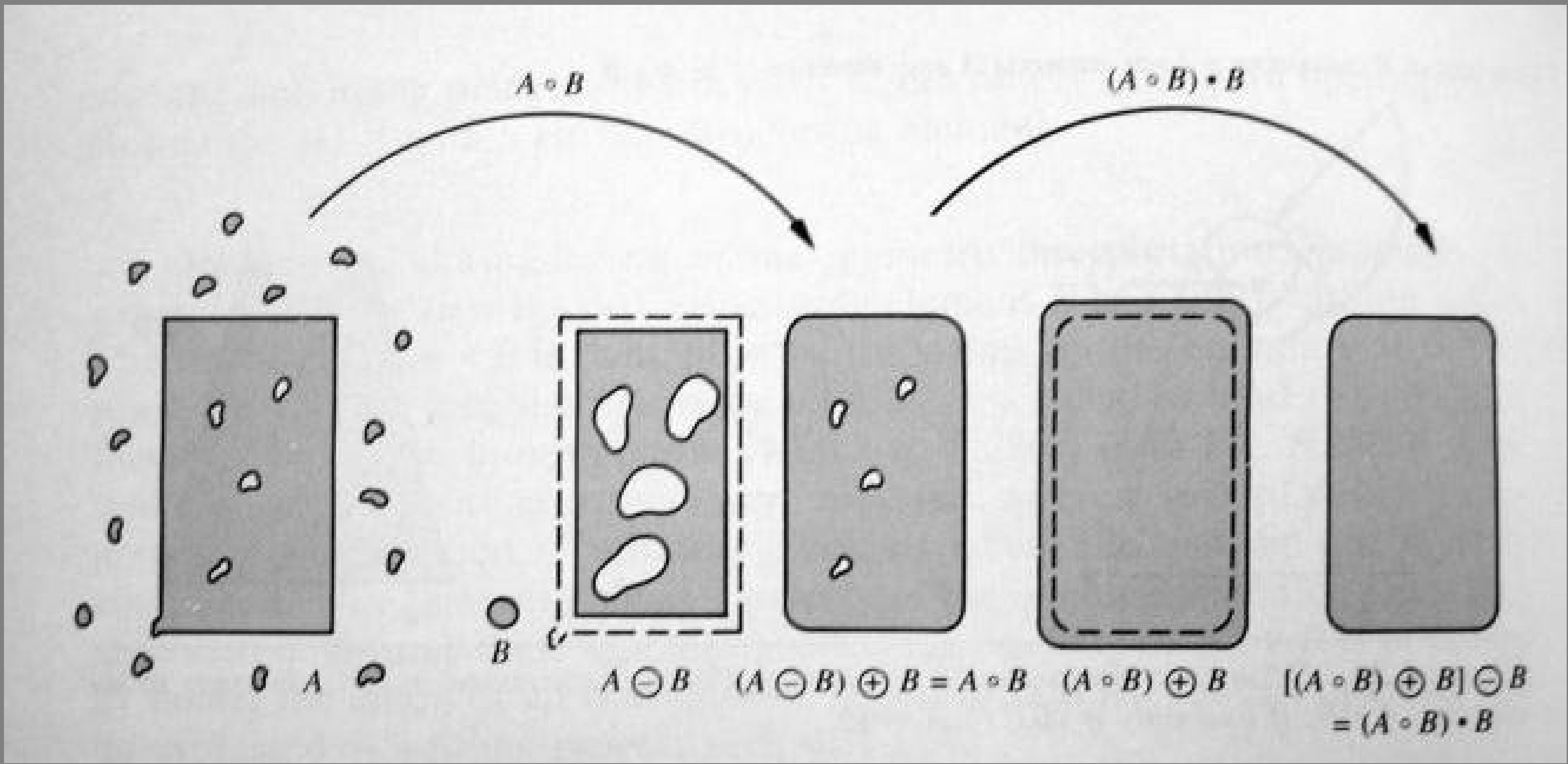


Image Preprocessing – An application of Opening/Closing

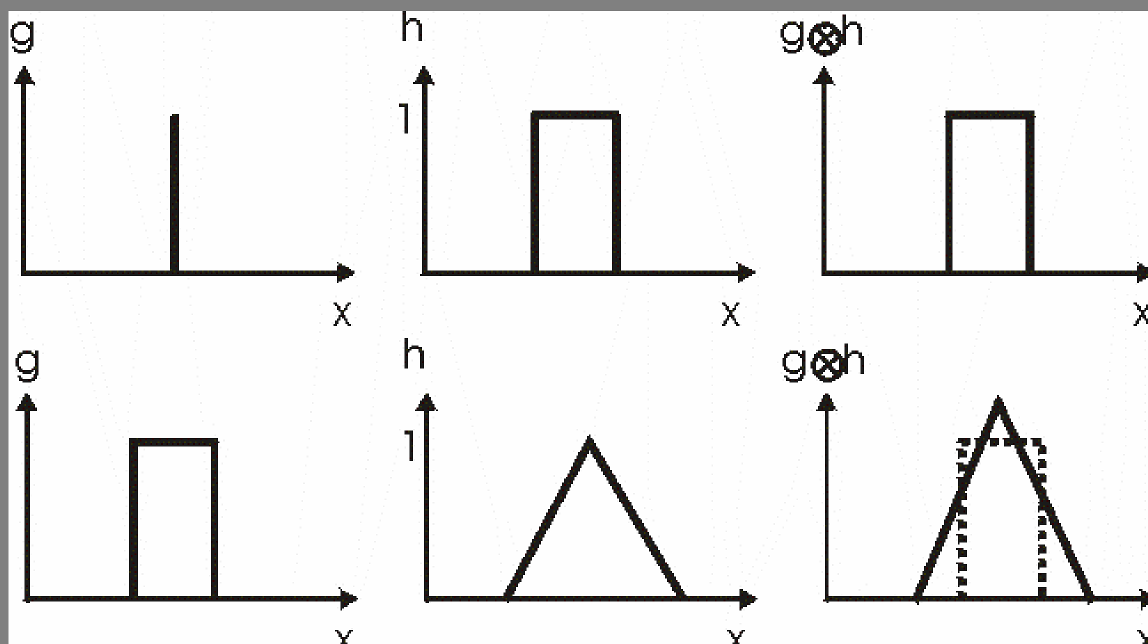


Convolution and Image Processing

Recall the definition of convolution:

$$g(t) \otimes h(t) = \int_{-\infty}^{\infty} g(t)h(t-t)dt$$

Graphical explanation of convolution:



Convolution Theorem

$$\Im(g \otimes h)(f) = \tilde{g}(f)\tilde{h}(f)$$

$$\int_{-\infty}^{\infty} g \otimes h(t) e^{-i2pft} dt = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(t)h(t-t)dt\, e^{-i2pft} dt$$

$$= \int_{-\infty}^{\infty} dt\, g(t) e^{-i2pft} \left(\int_{-\infty}^{\infty} dt\, h(t-t) e^{-i2pf(t-t)} \right)$$

$$= \int_{-\infty}^{\infty} dt\, g(t) e^{-i2pft} \left(\int_{-\infty}^{\infty} dt'\, h(t') e^{-i2pf(t')} \right)$$

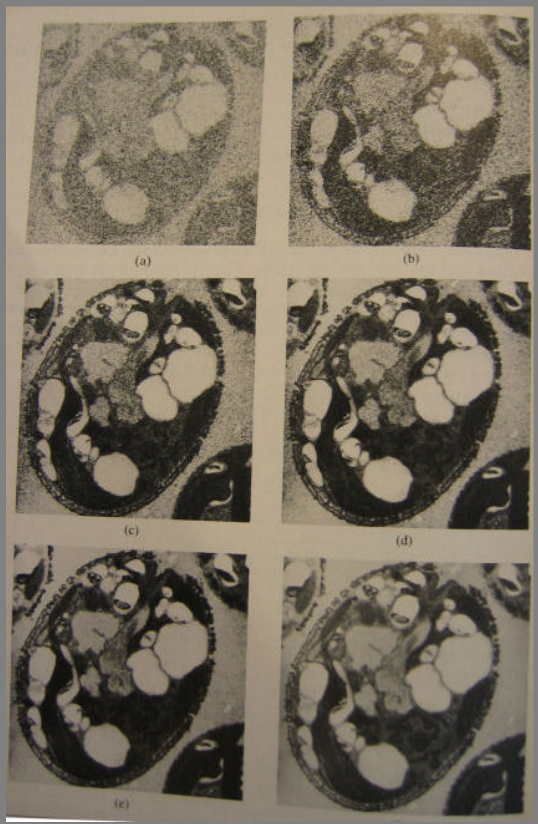
$$= \tilde{g}(f)\tilde{h}(f)$$

where $t' = t - t$ $\quad dt' = dt$

Image Preprocessing –
Noise Reduction, averaging, low pass filter, median filter

Averaging M images reduce noise by sqrt(M)

$$s_M = \frac{1}{\sqrt{M}} s_1$$



Avg by 2,8,16,32,128

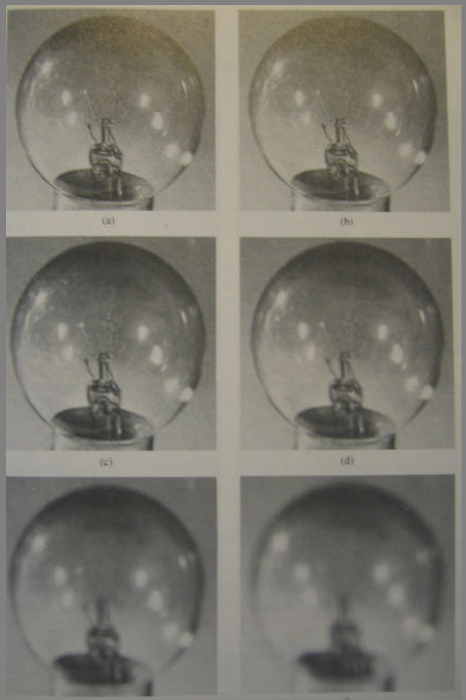
Low Pass Filter

3x3 kernel

$$\frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

5x5 kernel

$$\frac{1}{25}\begin{bmatrix}1&1&1&1&1\\1&1&1&1&1\\1&1&1&1&1\\1&1&1&1&1\\1&1&1&1&1\end{bmatrix}$$



Kernel 3,5,7,15,25

Median Filter

Replace center pixel value by the median value from a nxn pixel neighorhood

$$\begin{bmatrix}7&12&10\\6&9&12\\8&10&200\end{bmatrix}$$

Avg = 30; Median =10

5x5 low pass vs median

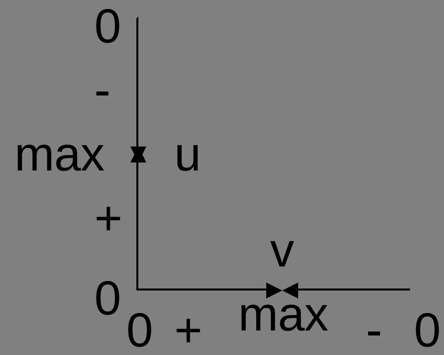
Image Preprocessing – Fourier Filtering I

2D Fourier Transform

$$F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\exp[-2pi(ux+vy)]dxdy$$

Power Spectrum

$$P(u,v) = \left|F(u,v)\right|^2$$



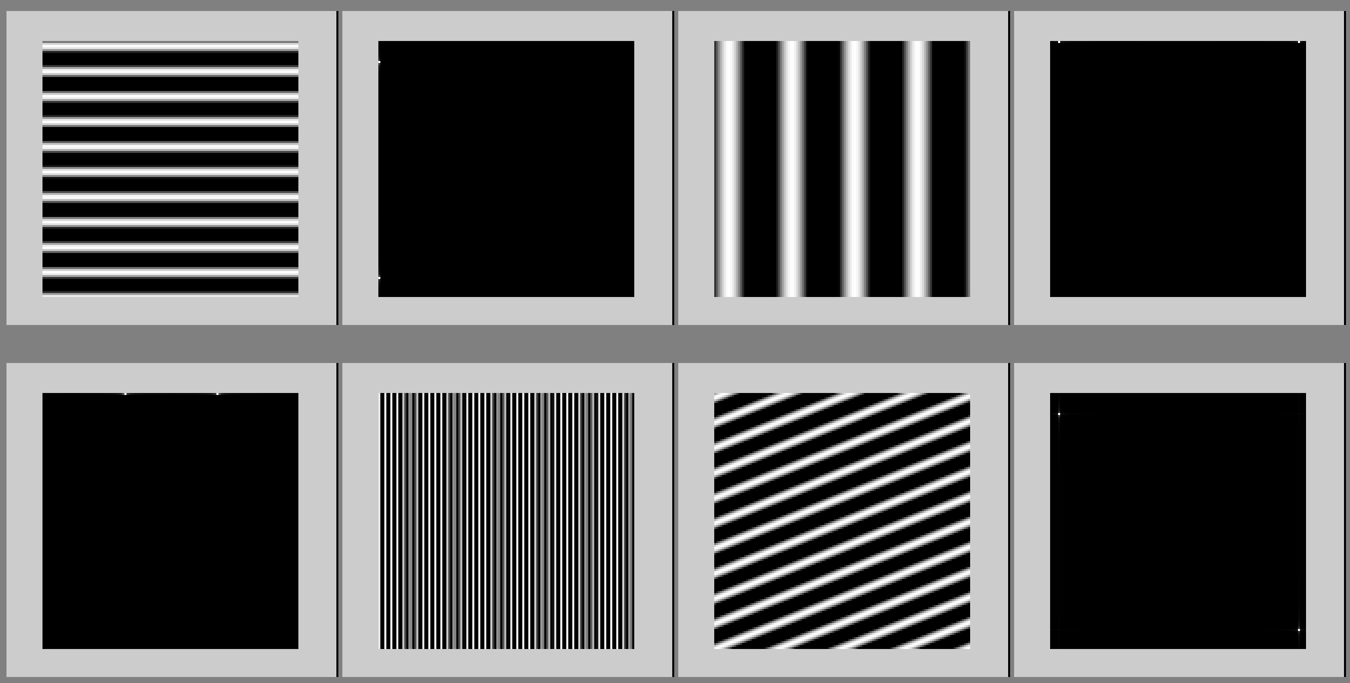
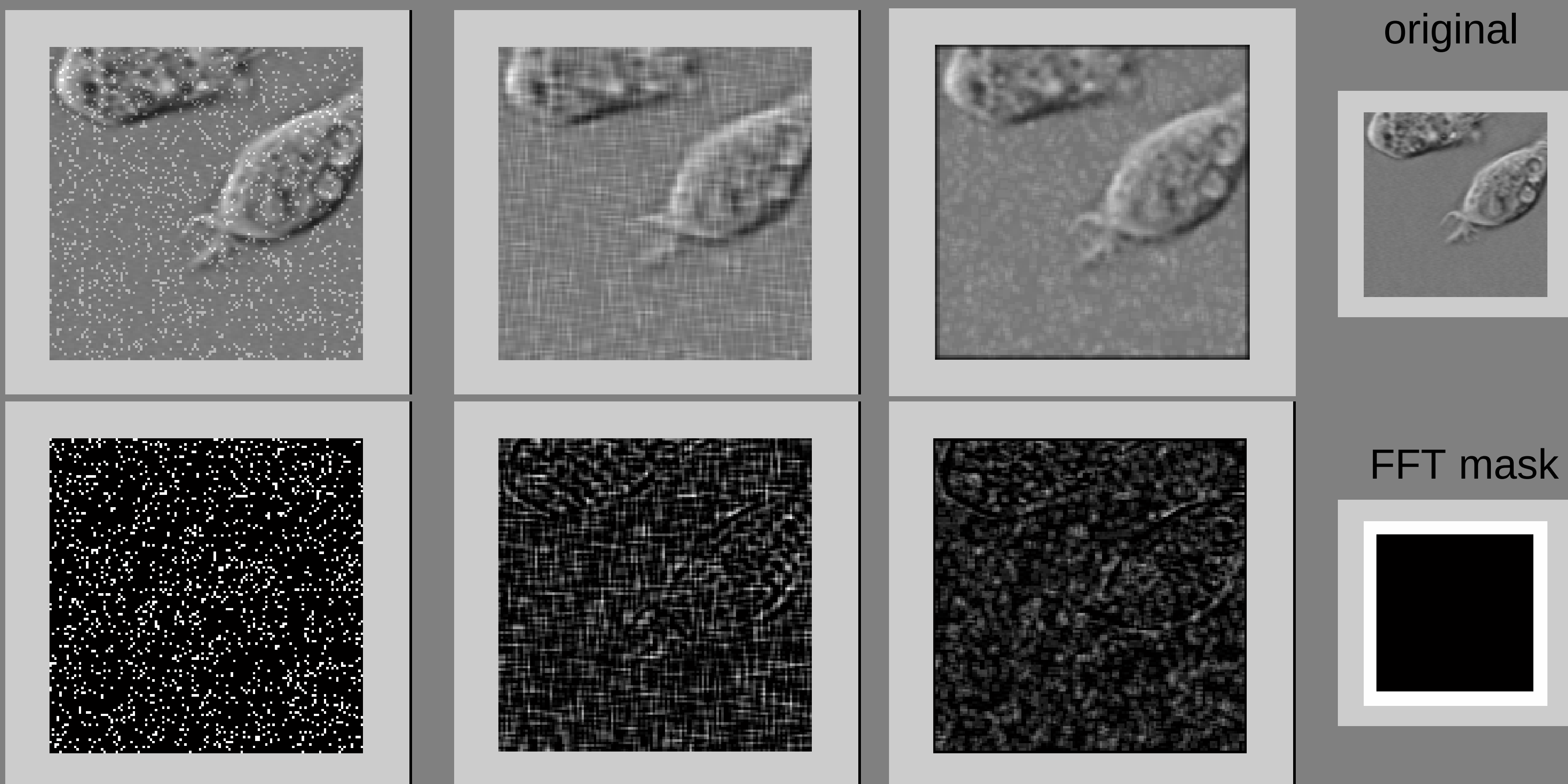


Image Preprocessing – Fourier Filtering II

Noise added | FFT low pass | Convolution $\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\1 & 1 & 1\\1 & 1 & 1\end{bmatrix}$



VAR= 100 | 41 | 33

Image Preprocessing – Fourier Filtering III

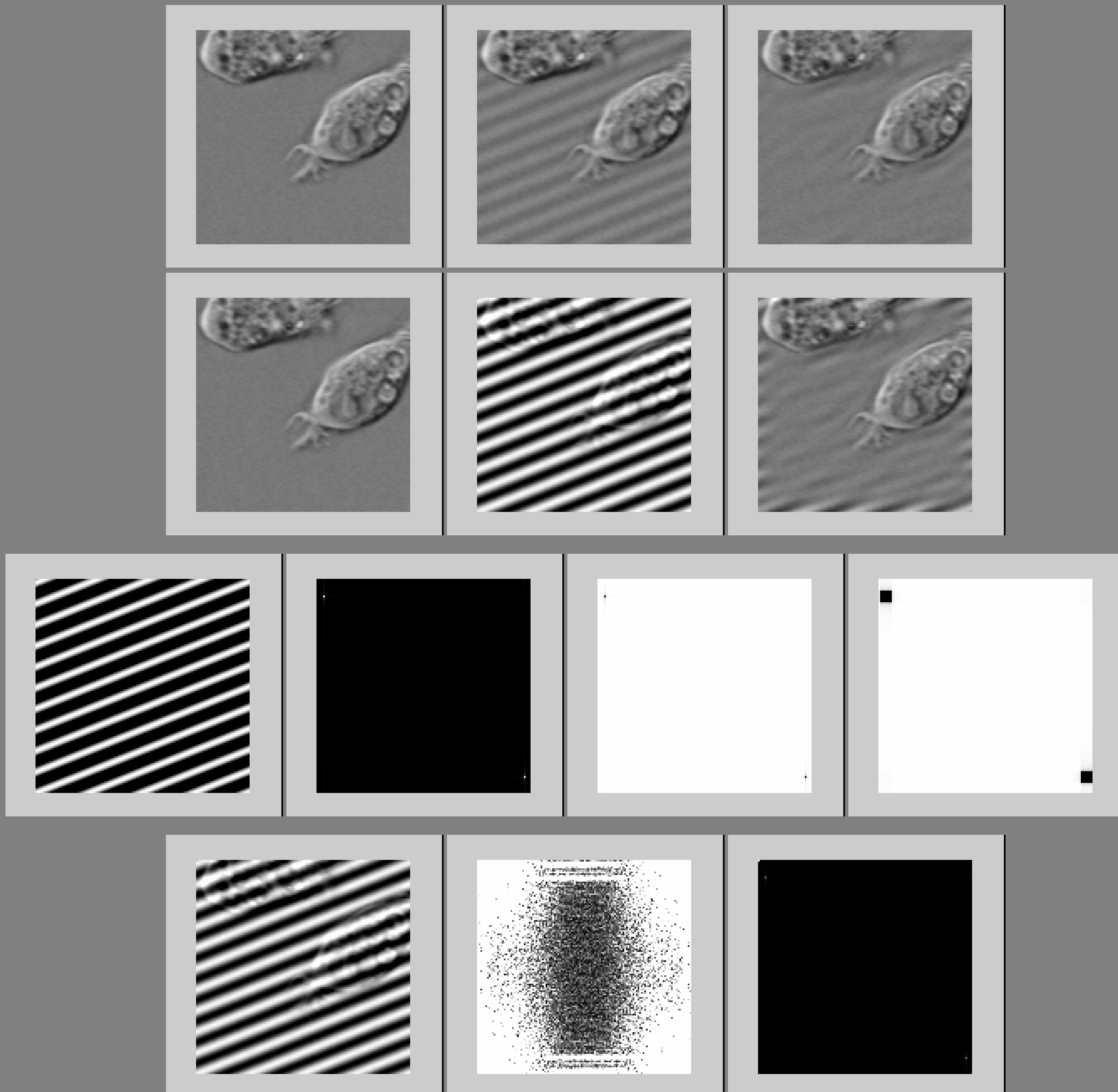
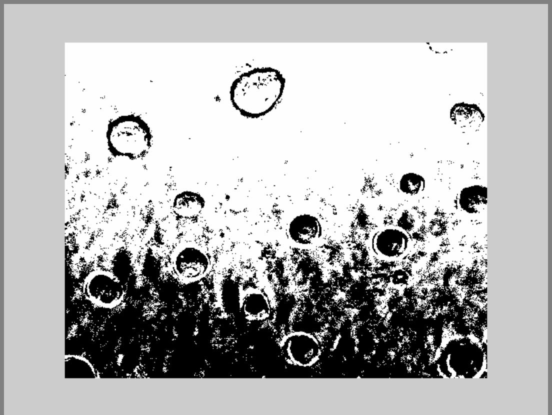
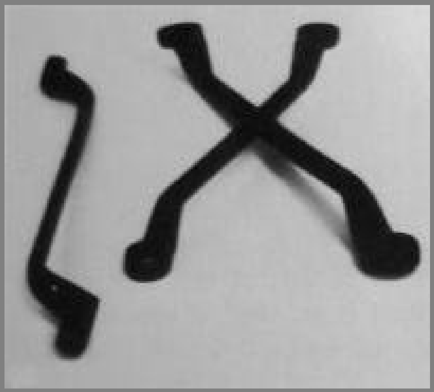


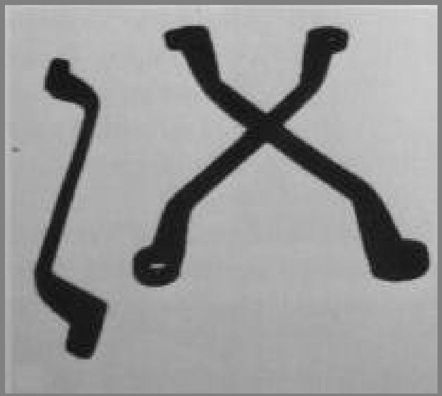
Image Segmentation –
Global Threshold, Optimal Threshold

Simple global threshold: $$N(i, j) = \begin{cases} 1 & if \;\; O(i, j) > T \\ 0 & if \;\; O(i, j) \leq T \end{cases}$$



Optimal thresholding





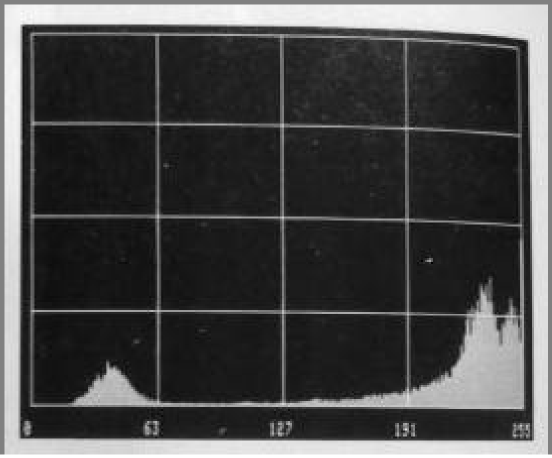


Image Segmentation – Edge Detection I

The edge is the boundary of two region with distinct intensity levels

The basic idea of edge detection is to compute the local derivative operator

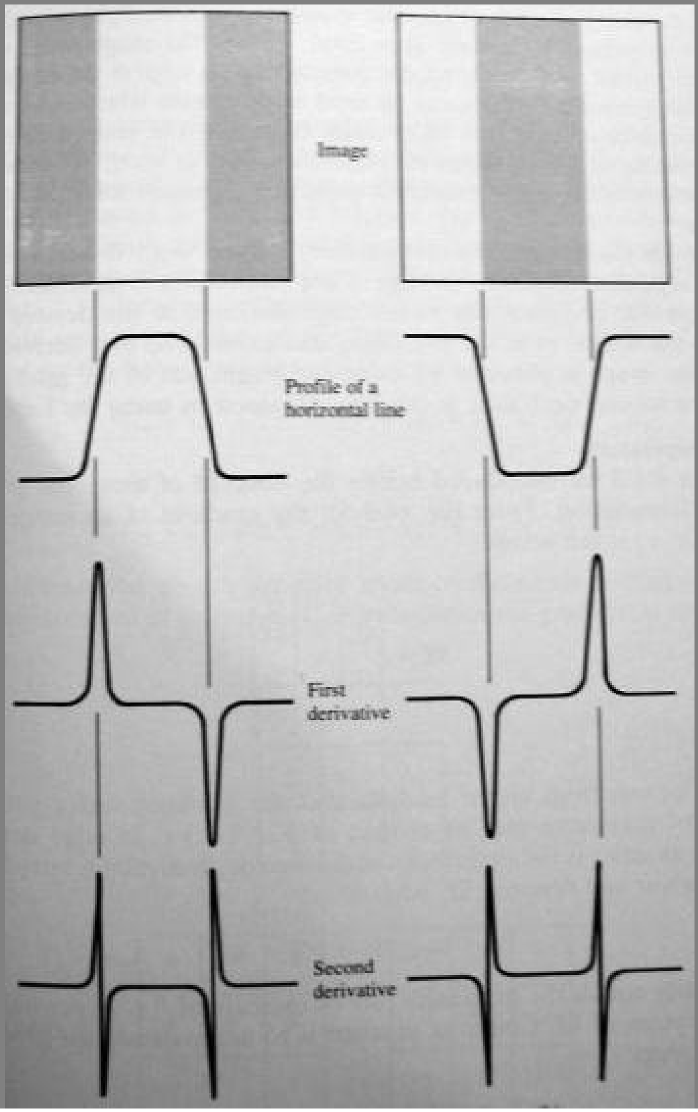


Image Segmentation – Edge Detection II

Gradient at each point (x,y) of an image

$$\nabla\vec{f} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

Magnitude $\nabla f = mag(\nabla\vec{f}) = (G_x^2 + G_y^2)^{1/2} \approx |G_x| + |G_y|$

Direction $a = \tan^{-1}(\frac{G_y}{G_x})$

Sobel Implementation

$$G_x = (z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7) \qquad G_y = (z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)$$

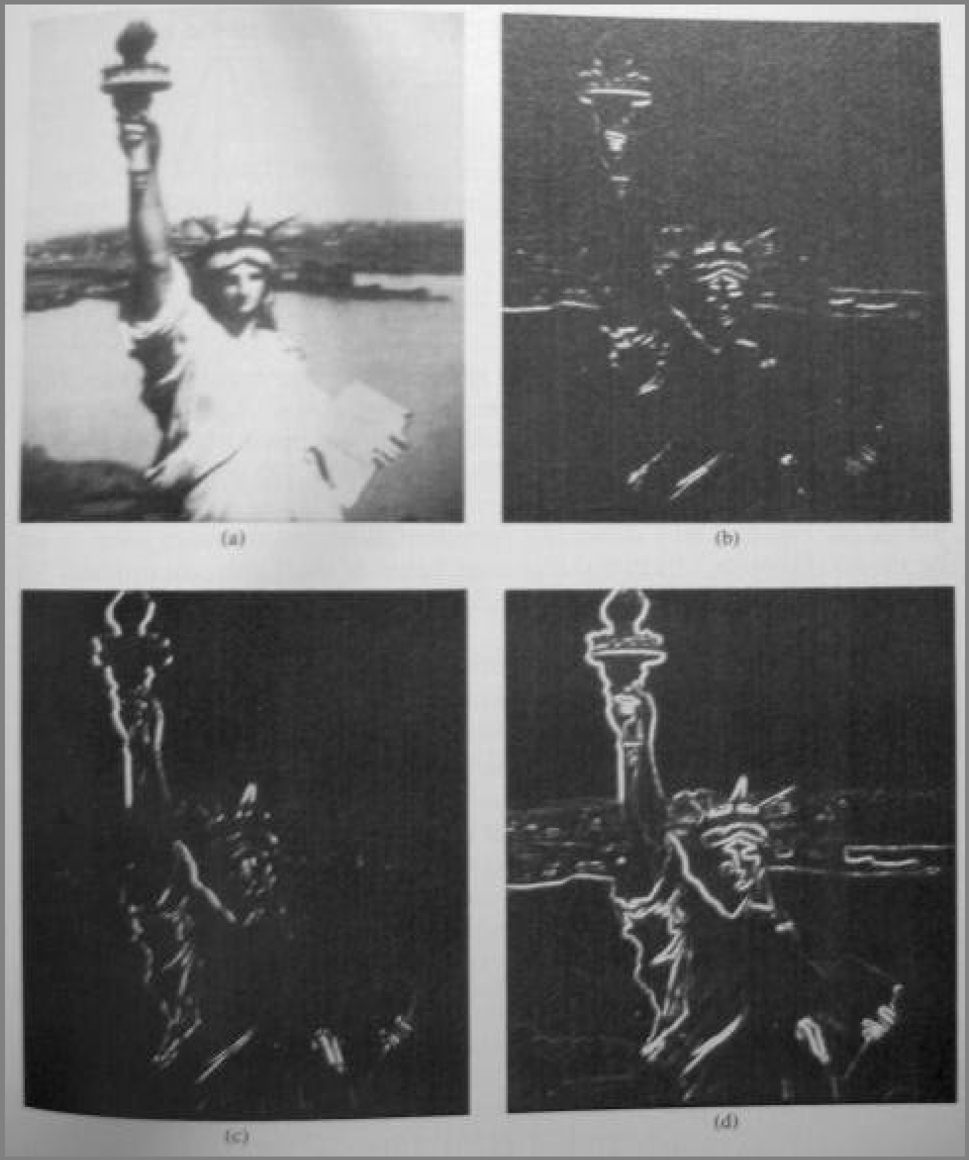
$$\begin{bmatrix} z_1 & z_2 & z_3 \\ z_4 & z_5 & z_6 \\ z_7 & z_8 & z_9 \end{bmatrix}$$

Gx $\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$ Gy $\begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$

Laplacian at each point (x,y) of an image

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 4z_5 - (z_2 + z_4 + z_6 + z_8)$$

Image Segmentation – Edge Detection III



Gy

Gx

$$\left|G_x\right| + \left|G_y\right|$$

Image Segmentation – Boundary based Threshold and Region Filling

(1) Identify region based on first finding the boundaries:

$$s(x,y)=\begin{cases} 0 & \text{if } \nabla f < T \\ + & \text{if } \nabla f > T \text{ and } \nabla^2 f \geq 0 \\ - & \text{if } \nabla f > T \text{ and } \nabla^2 f < 0 \end{cases}$$

Creates a 3 level image based on gradient and Laplacian operators. Boundaries at transition of (-,+) and (+,-)

(2) Edge Linking

After boundary pixels are identified. Due to noise, a linking procedure is often needed. Linking can be accomplished by closing operation or regional growing algorithms (discussed later)