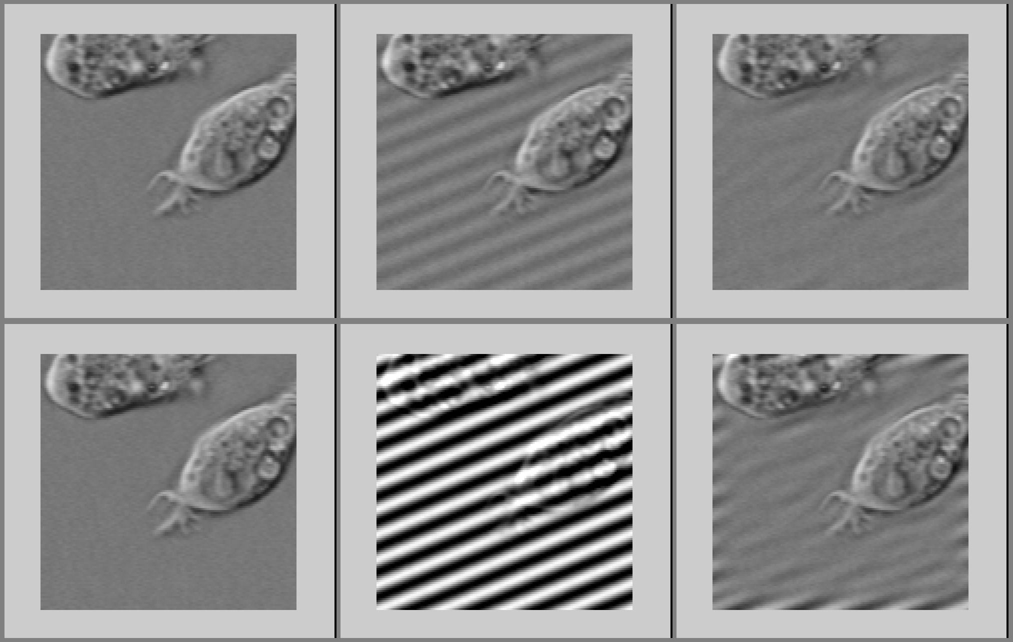
Image Processing and Analysis II



Materials extracted from Gonzalez & Wood
and Castleman

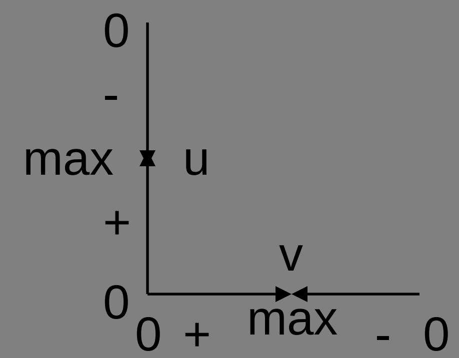
Image Preprocessing – Fourier Filtering I

2D Fourier Transform

$$F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\exp[-2\pi i(ux+vy)]dxdy$$

Power Spectrum

$$P(u,v) = \left|F(u,v)\right|^2$$



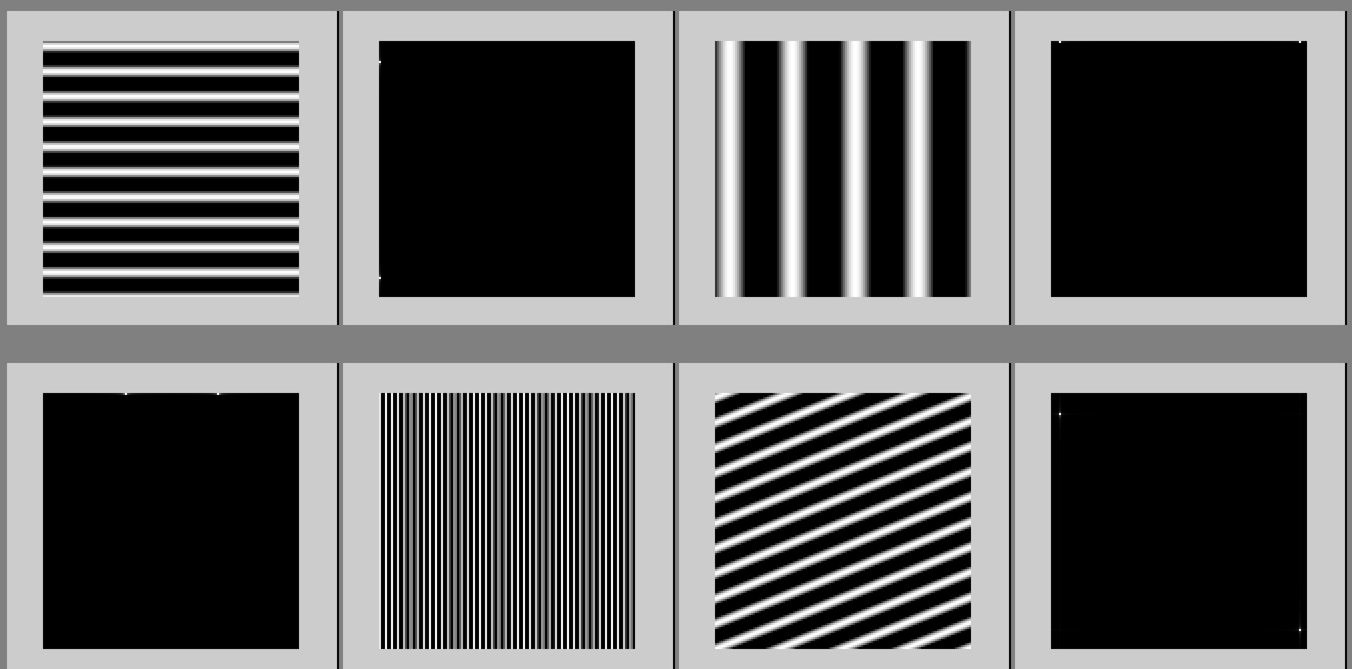
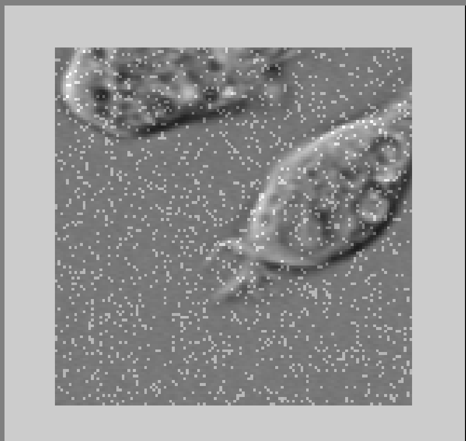
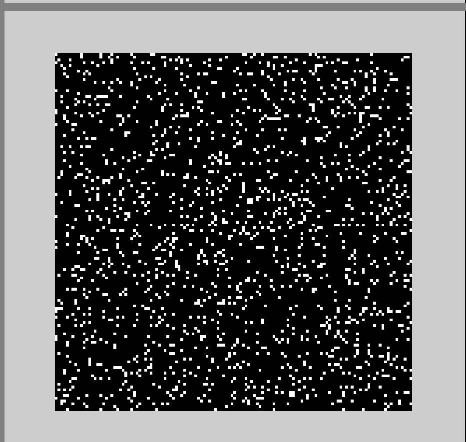


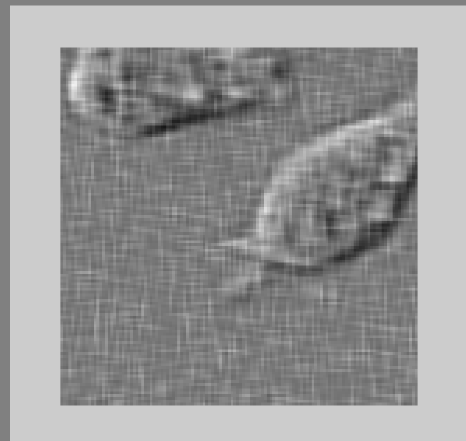
Image Preprocessing – Fourier Filtering II

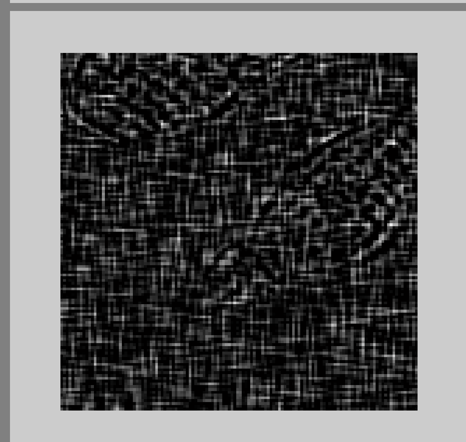
Noise added



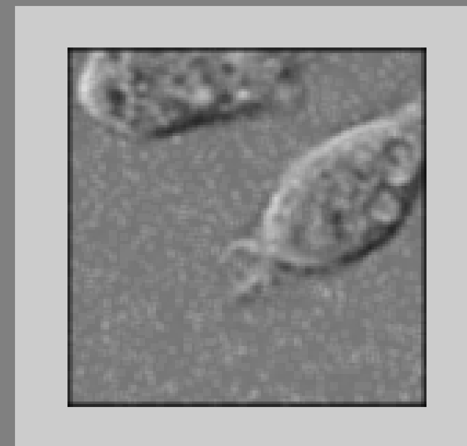


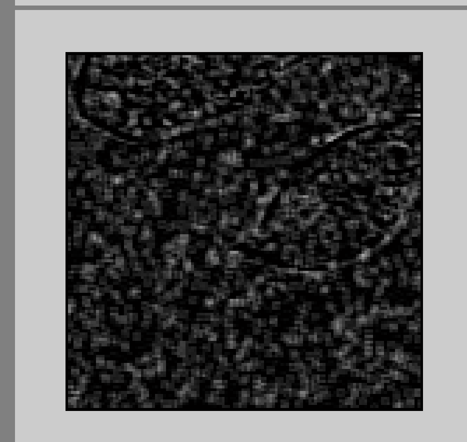
FFT low pass





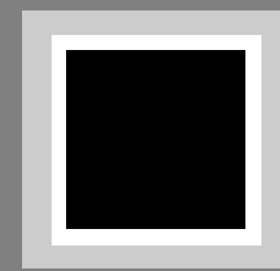
Convolution $\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$





original

FFT mask



VAR= 100

41

33

Image Preprocessing – Fourier Filtering III

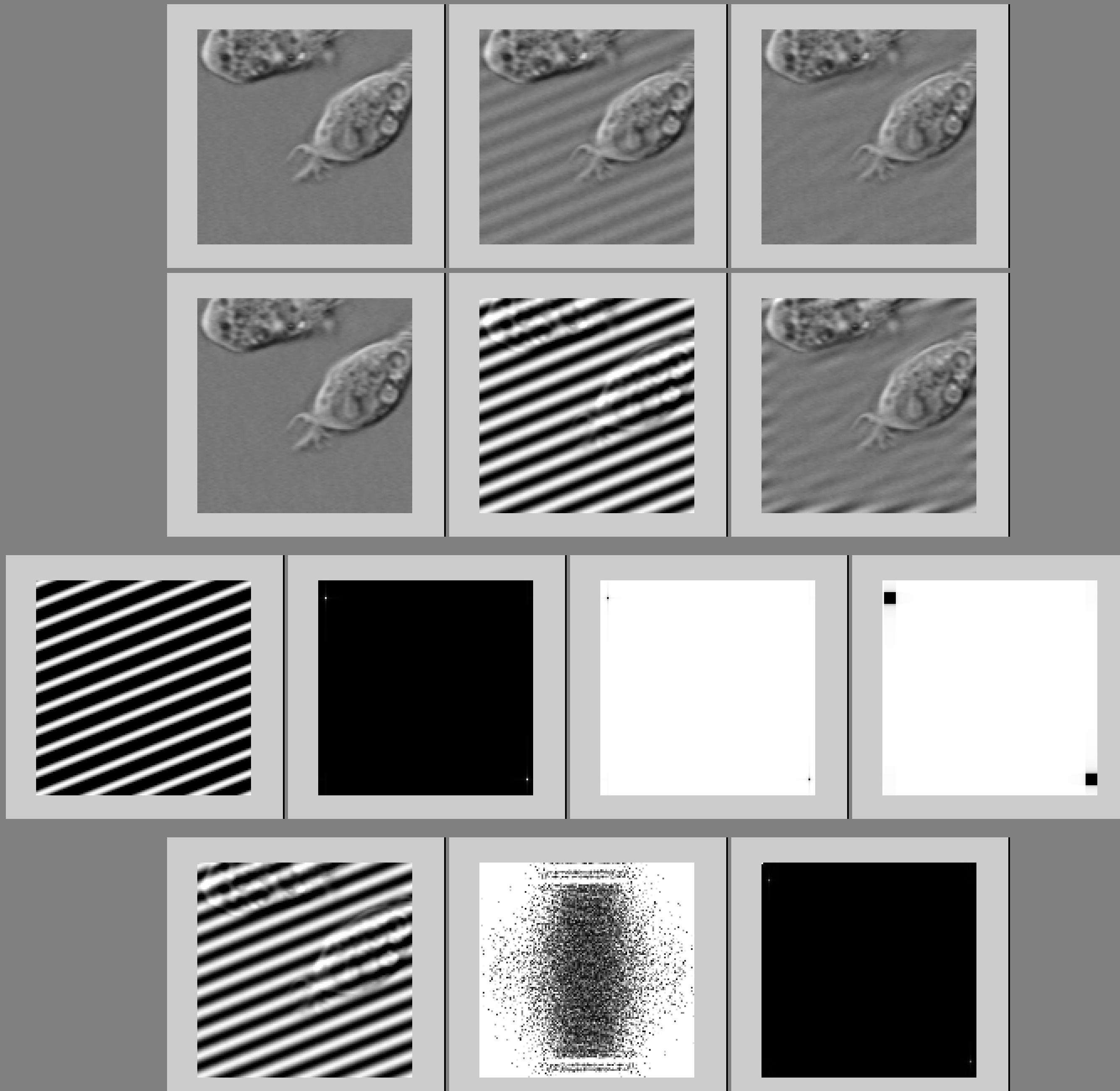
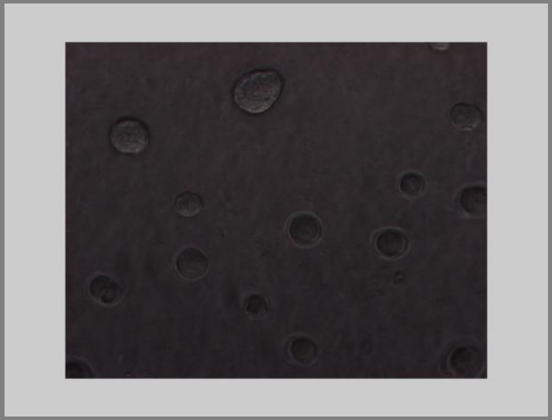
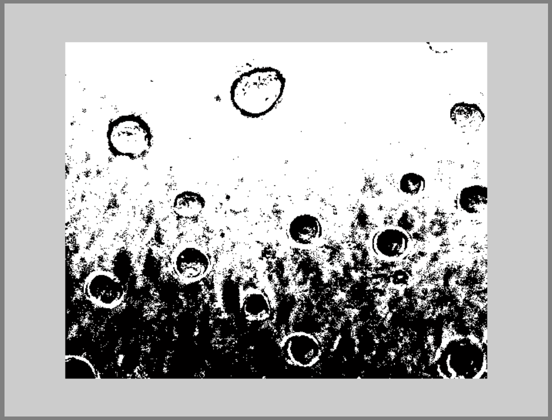


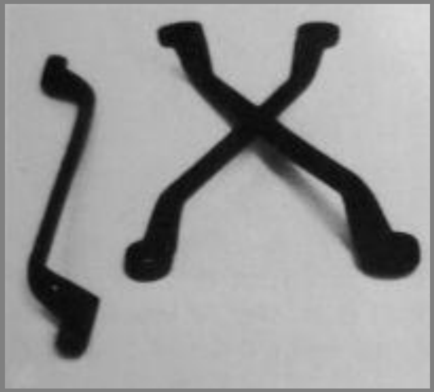
Image Segmentation – Global Threshold, Optimal Threshold

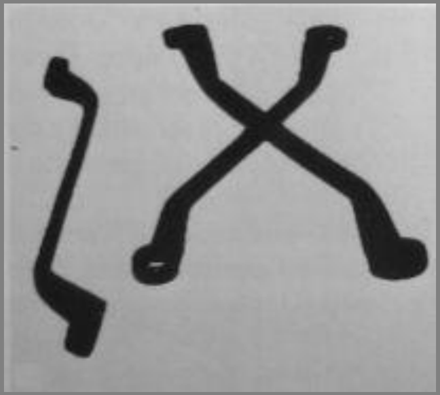
Simple global threshold: $$N(i,j) = \begin{cases} 1 & if\ \ O(i,j) > T \\ 0 & if\ \ O(i,j) \le T \end{cases}$$





Optimal thresholding





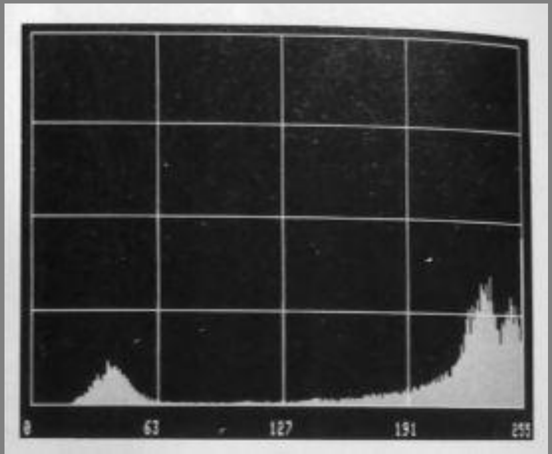


Image Segmentation – Edge Detection I

The edge is the boundary of two region with distinct intensity levels

The basic idea of edge detection is to compute the local derivative operator

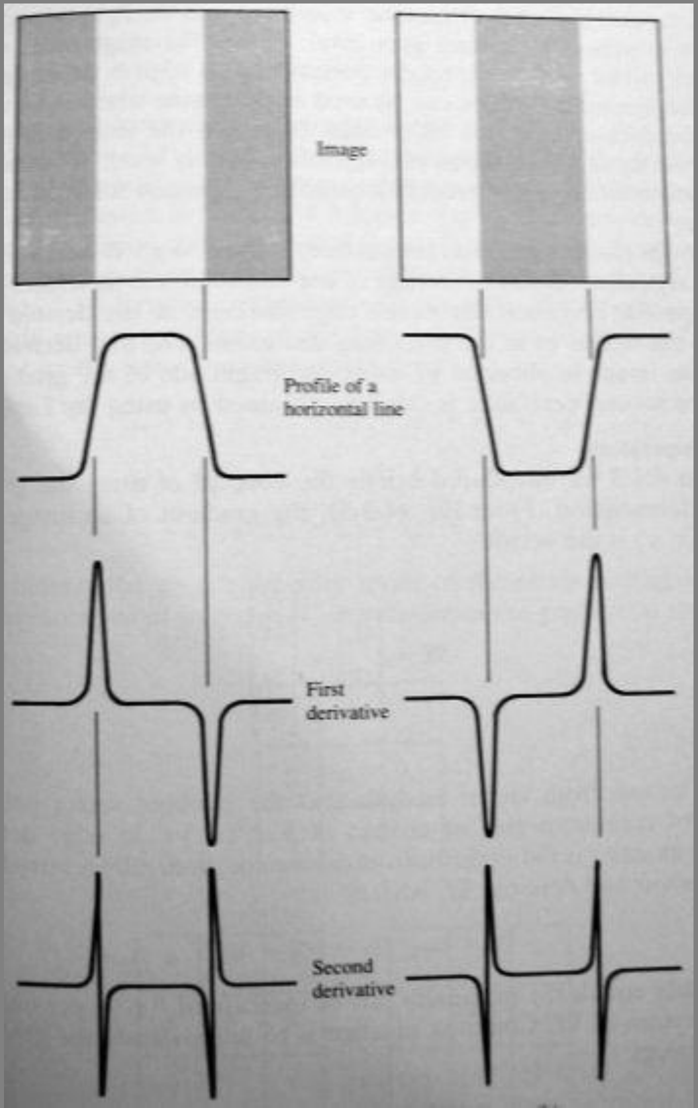


Image Segmentation – Edge Detection II

Gradient at each point (x,y) of an image

$$\nabla\vec{f} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

Magnitude $\nabla f = mag(\nabla\vec{f}) = (G_x^2 + G_y^2)^{1/2} \approx |G_x| + |G_y|$

Direction $\alpha = \tan^{-1}(\frac{G_y}{G_x})$

Sobel Implementation

$$G_x = (z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7) \qquad G_y = (z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)$$

$$\begin{bmatrix} z_1 & z_2 & z_3 \\ z_4 & z_5 & z_6 \\ z_7 & z_8 & z_9 \end{bmatrix}$$

Gx $\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$

Gy $\begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$

Laplacian at each point (x,y) of an image

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 4z_5 - (z_2 + z_4 + z_6 + z_8)$$

Image Segmentation – Edge Detection III

Gy

Gx

$$\left|G_x\right| + \left|G_y\right|$$

Image Segmentation – Boundary based Threshold and Region Filling

(1) Identify region based on first finding the boundaries:

$$s(x,y) = \begin{cases} 0 & \text{if } \nabla f < T \\ + & \text{if } \nabla f > T \text{ and } \nabla^2 f \geq 0 \\ - & \text{if } \nabla f > T \text{ and } \nabla^2 f < 0 \end{cases}$$

Creates a 3 level image based on gradient and Laplacian operators. Boundaries at transition of (-,+) and (+,-)

(2) Edge Linking

After boundary pixels are identified. Due to noise, a linking procedure is often needed. Linking can be accomplished by closing operation or regional growing algorithms (discussed later)

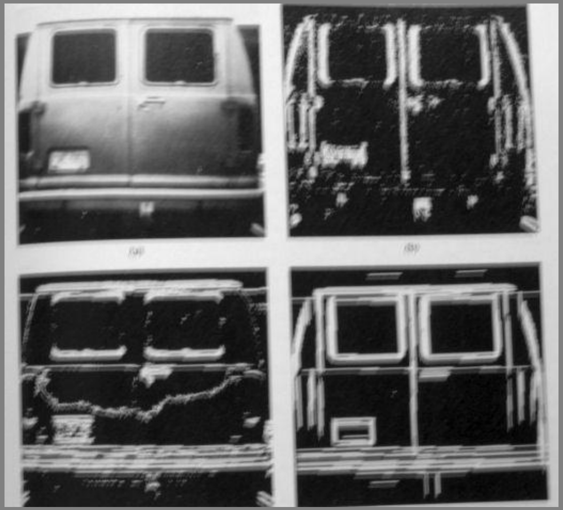


Image Preprocessing – Morphological Operations I

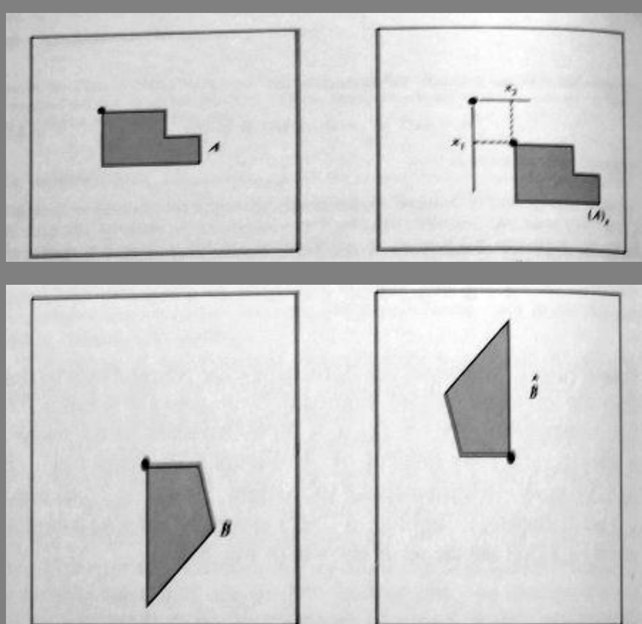
Let A and B be a set of points in the image.

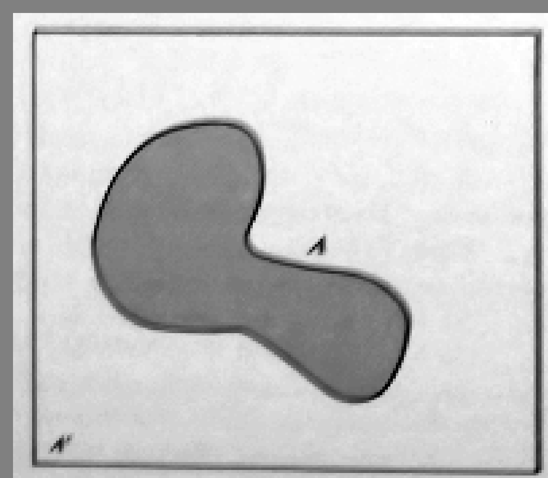
Translation of A by a vector x: $(A)_x = \{c \mid c = a + x, \quad for\ a \in A\}$

Reflection of B: $\hat{B} = \{x \mid x = -b, \quad for\ b \in B\}$

Complement of A: $A^c = \{x \mid x \notin A\}$

Difference of A & B: $A - B = \{x \mid x \in A\ and\ x \notin B\}$





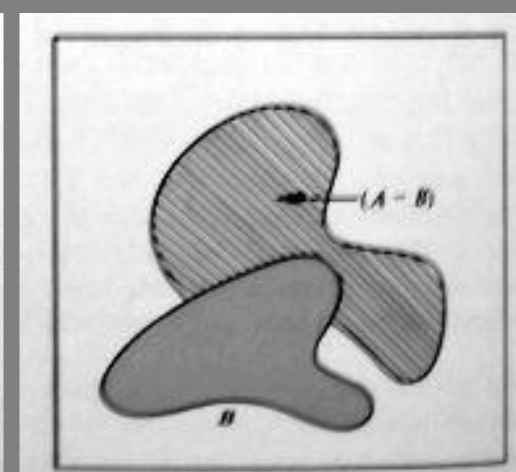
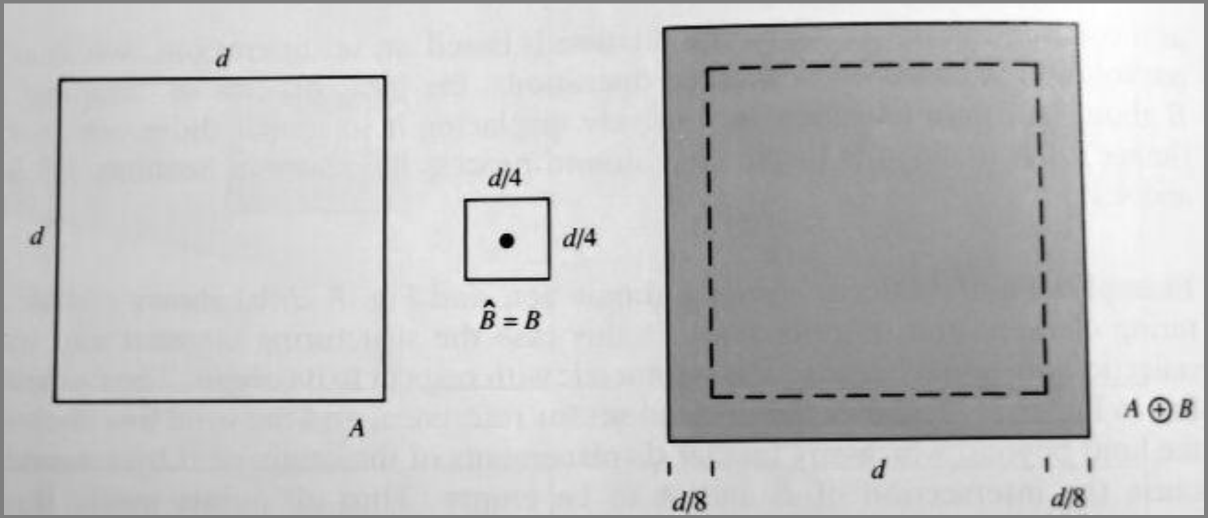


Image Preprocessing – Morphological Operations II

Diluation of A by B: $A \oplus B = \{x \mid [(\hat{B})_x \cap A] \neq \phi\}$

Erosion of A by B: $A \Theta B = \{x \mid (B)_x \subseteq A\}$

Note $(A \Theta B)^c = A^c \oplus \hat{B}$



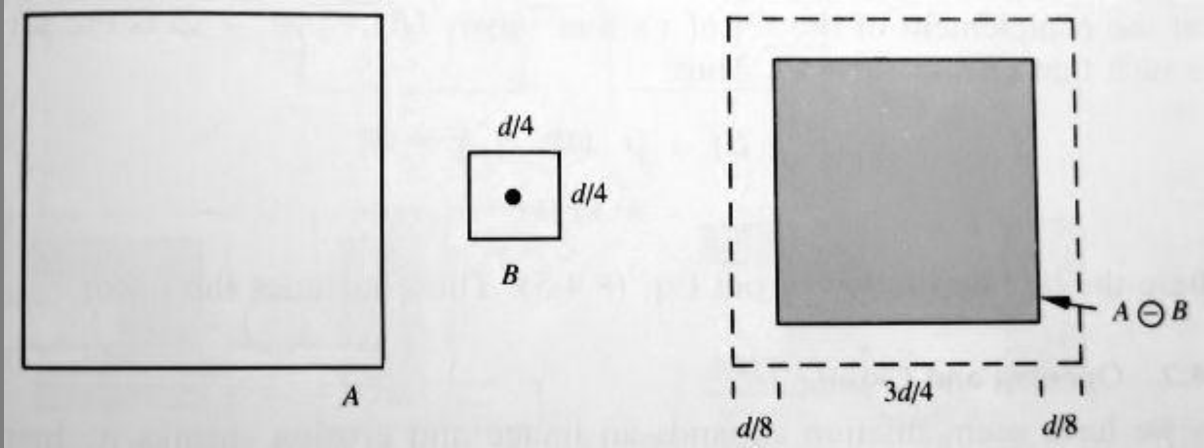
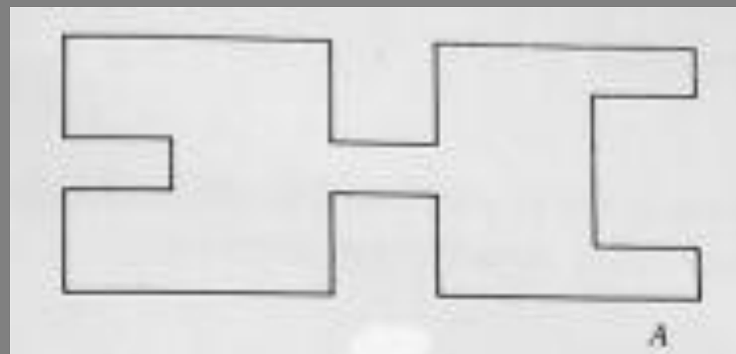


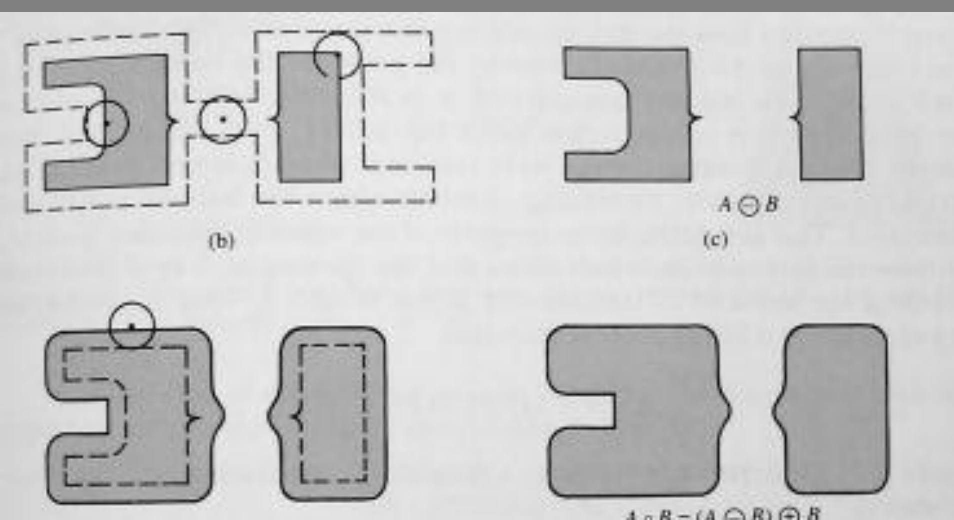
Image Preprocessing – Morphological Operations III

Opening of A by B: $A \circ B = (A \Theta B) \oplus B$

Closing of A by B: $A \bullet B = (A \oplus B) \Theta B$

Note $(A \circ B) \circ B = A \circ B$ and $(A \bullet B) \bullet B = A \bullet B$





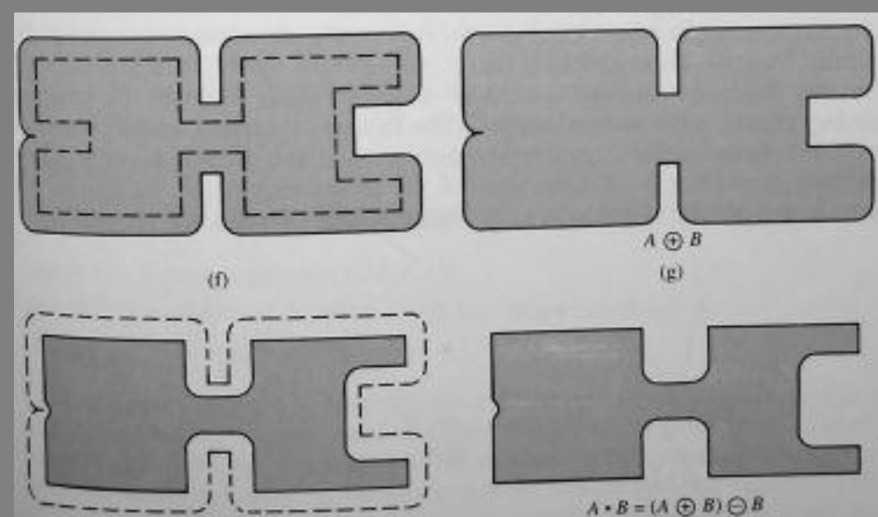


Image Preprocessing – An application of Opening/Closing

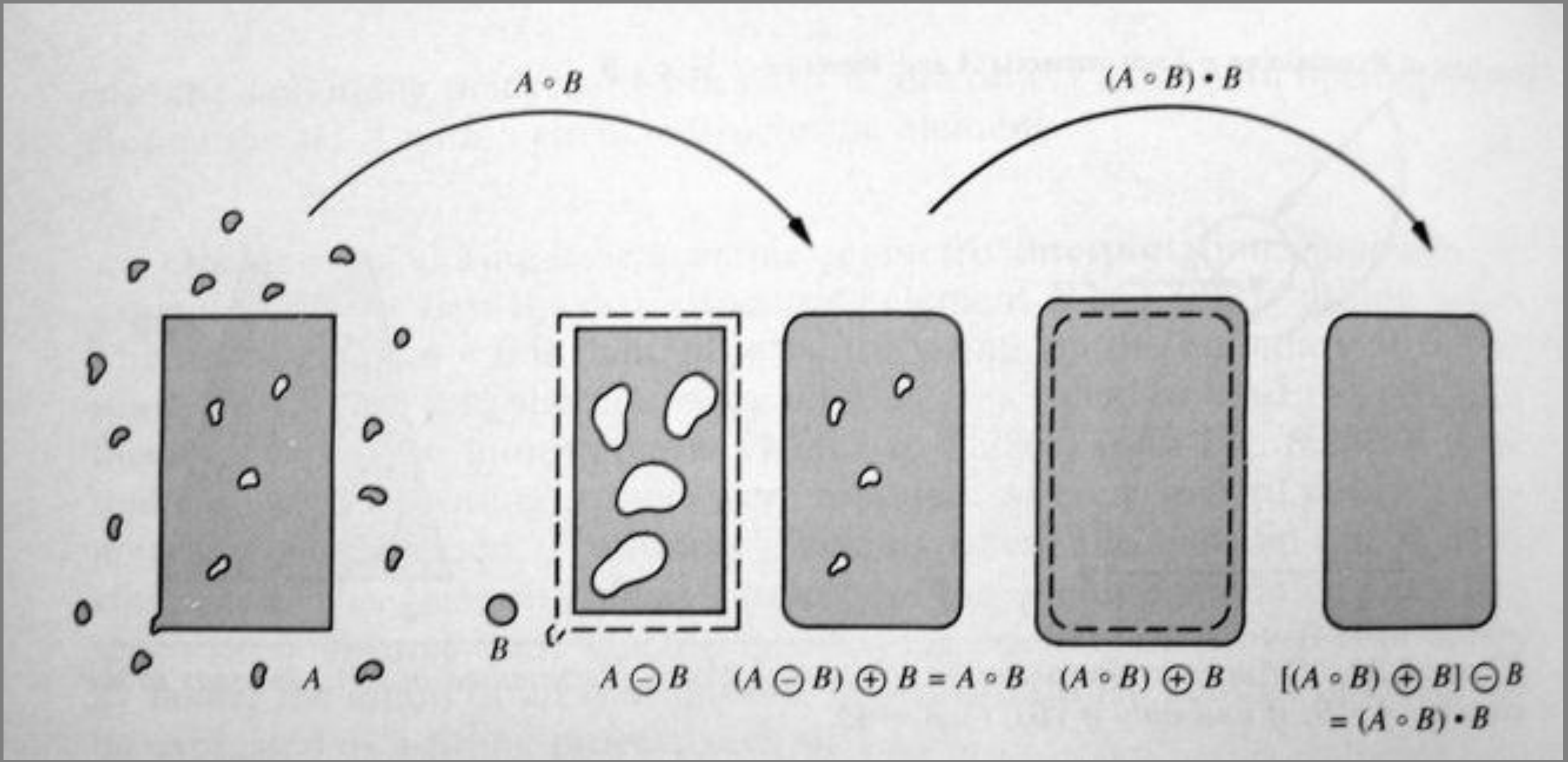


Image Segmentation –
Boundary based Threshold and Region Filling II

(3) Region Filling

Assign all boundary points to zero.
Identify a point P inside the boundary, the region can be filled by iterative application to neighboring points of:

$$X_k = (X_{k-1} \oplus B) \cap A^c$$

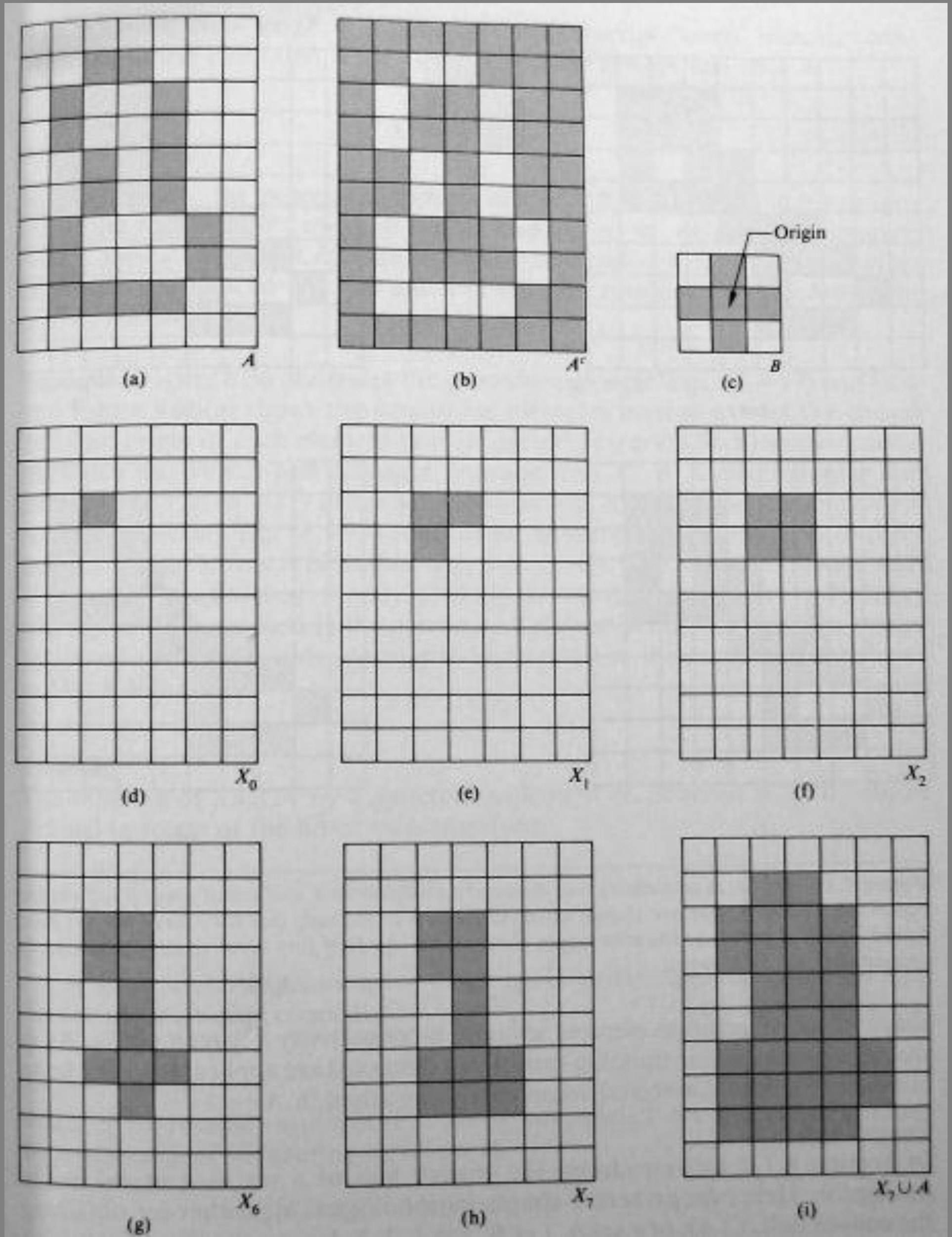


Image Segmentation – Region Growing by Pixel Aggregation

Basic idea: Selected a number of “seed” pixels in the image. Find neighbors that are similar in value. Aggregate similar value pixels into a region. Merge regions with similar values. Add more seeds as necessary until all the picture is filled.

	1	2	3	4	5
1	0	0	5	6	7
2	1	1	5	8	7
3	0	<u>1</u>	6	<u>7</u>	7
4	2	0	7	6	6
5	0	1	5	6	5

(a)

a	a	b	b	b
a	a	b	b	b
a	a	b	b	b
a	a	b	b	b
a	a	b	b	b

(b)

Threshold 3

a	a	a	a	a
a	a	a	a	a
a	a	a	a	a
a	a	a	a	a
a	a	a	a	a

(c)

Threshold 8

Image Classification and Recognition I

Image recognition is the problem of classifying patterns. Pattern classes can be Denoted by M classes: $\omega_1, \omega_2, \omega_3 \ldots \omega_M$

Recognition problem is relatively straightforward if each class can be distinctly described by some measurable characteristics denoted by the pattern vector x={x1, x2, x3, ….}

Example, classify images of three type of iris flowers (setosa (ω_1), virginica (ω_2), and versicolor (ω_3)) by their petal width (x1) and petal length (x2)

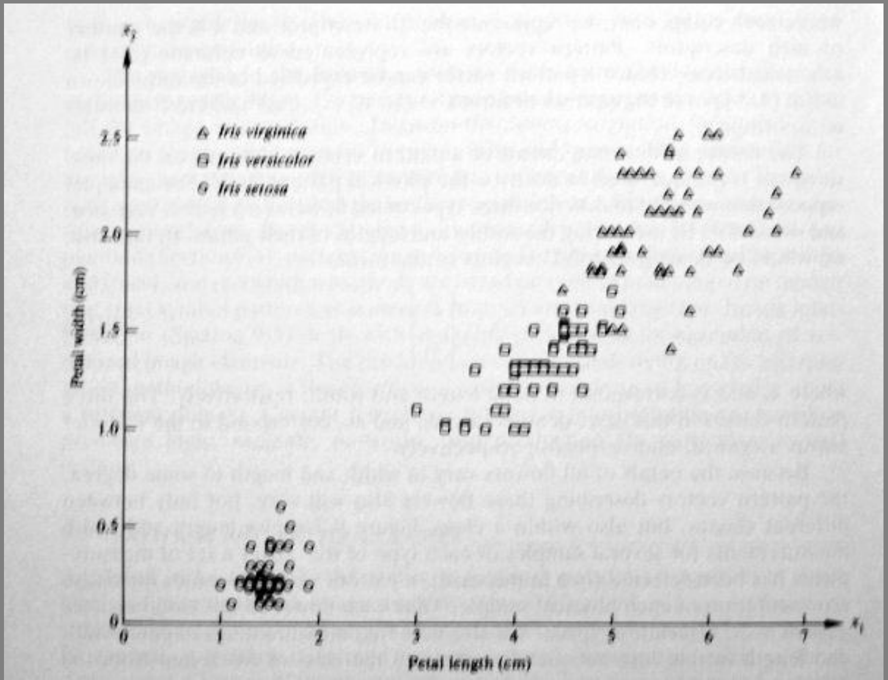


Image Classification and Recognition II

We can define a n dimensional characteristic vector for each class i:

$$\vec{X}_i = \{X_i^1 \cdots X_i^n\}$$

We can define M distances of a pattern found in the image to each defined class:

$$d_i = \sqrt{\sum_j^n (x^j - X_i^j)^2}$$

Where $\vec{x} = \{x^1 \ldots x^n\}$ is the pattern vector of the pattern in question

Then the pattern before to class ω_i if:

$$d_i(\vec{x}) < d_j(\vec{x}) \quad j = 1,2,\ldots,M;\ j \neq i$$