Electronically Measuring the Flight Time of Light

Author: Jesse J. Smith
Experimentalists: Jesse J. Smith and Kyle Martin

Physics and Astronomy Department, University of New Mexico,
Albuquerque, NM 87131

Physics 307L Junior Lab: November 7, 2007
smithjj@unm.edu

Abstract. Attempting to measure the speed of light, we used a time amplitude converter to measure the time between the emission of a light signal by a light emitting diode and the detection of the light signal by a photomultiplier tube. The slope of a line fit using the least-squares method of many measurements taken while varying the distance the light signal travels should approximate the speed of light. Our measurement of $(3.06 \pm 0.18) \times 10^8$ meters per second is in good agreement with the accepted value of 2.998×10^8 meters per second.

1 Introduction

Every form of electromagnetic radiation travels through a vacuum at the same speed, regardless of frequency or wavelength. In 1905, Albert Einstein proposed in his theory of special relativity that this speed was even constant regardless of the frame of the observer relative to the source, provided the reference frames are inertial. The speed of light in a vacuum is also the fastest ordinary objects with mass can travel. The permittivity of free space and the magnetic constant (which appear frequently in electromagnetism) are also related to the speed of light by the equation $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$, where ϵ_0 is the permittivity of free space and μ_0 is the magnetic constant.

In 1983, the value of the meter was redefined to make the speed of light exactly 299,792,458 meters per second (Tholen et al., 1983). Historically, however, the speed of light was one of the most studied - and measured - physical constants in science.

Background

Aristotle, an ancient Greek philosopher circa 350 BC, and Heron of Alexandria, an ancient Greek physicist and mathematician circa 60 AD, believed the speed of light to be infinite; that is, light reached its destination at the very instant it was emitted (Wikipedia, 2007). Early attempts at measuring the speed of light, while not very accurate or precise, proved that it was finite.

Several methods of measuring the speed of light produced astoundingly accurate results in the latter half of the 19th century and the early 20th century. These methods involved measurements of the speed of light propagating through air; this speed is very close to the speed of light through a vacuum, as the refractive index (the ratio of the speed of light through a vacuum to the speed of light through a given medium) of air is 1.0003.

Hippolyte Fizeau's attempt in 1849 used a rotating, notched wheel and a mirror thousands of meters away from a light source. Light shone on the rotating wheel and struck the mirror only when the wheel's cogs were not blocking it. The mirror reflected the light back at the rotating wheel, and an observer near the light source would detect the reflected light only when the wheel did not block it on its second pass, which occurred only at specific speeds of rotation. The speed of light through air could then be calculated, given this speed, the number of teeth on the wheel and the distance between the light source, mirror and observer. Fizeau concluded the speed of light must be around 313,000 kilometers per second (Fizeau, 1849).

Several subsequent improvements boosted the accuracy and precision of this method. Leon Foucalt replaced the rotating wheel by a rotating mirror, and in 1862 published the results of his measurement: 298,000 kilometers per second. Albert A. Michelson devoted much of his career to measuring the speed of light to great precision; in 1935, he used a rotating prism and a mirror more than 20 miles from a light source to measure the speed of light to be 299,794 $\pm$ 11 kilometers per second (Michelson et al., 1935).

After World War II, Louis Essen and A.C. Gordon-Smith used a microwave cavity to measure the speed of light. Their conclusion of 299,792 $\pm$ 3 kilometers per second was refined to 299,792.5 $\pm$ 1 kilometers per second by 1950 (Essen & Gordon-Smith, 1948).

In this report, we used a time amplitude converter (TAC) to measure the time of flight of a light pulse from a light emitting diode to a photomultiplier tube. The TAC outputs a voltage proportional to the time delay between triggering events. The TAC is triggered by a capacitor discharging which fires the light emitting diode and once again when the photomultiplier tube registers a drop in voltage caused by the incident light pulse. The speed of light is measured by finding the slope of a line fit by using the least squares method of measurements taken varying the distance light pulses must travel.

2 Methods and Materials

Required equipment

- Time Amplitude Converter (TAC): Model 567 mfd. by EG&G Ortec
- Delay Module: nSec Delay model 2058 mfd. by Canberra
- Digital Storage Oscilliscope (DSO): Tektronics TDS 1002 (Dual channel digital storage oscilloscope)
- LED Power Supply: Model 6207a mfd. by Harrison Industries (DC power supply, 0-200V, 0-0.2A)
- LED/capacitor module: Unknown manufacturer. Consists of a green LED and timer circuit. Cycles on and off at around 10KHz, depending on the voltage applied, which is around 200 volts DC
- Photomultiplier Tube (PMT): Labeled N-134, unknown manufacturer, with magnetic shielding tube attached to the front of it
- PMT Power Supply: Model 315 mfd. by Bertan Associates, Inc. (DC power supply 0-5000V, 0-5mA)
- Long cardboard tube, about 15 centimeters in diameter and 5 meters long
- 3 meter sticks taped together

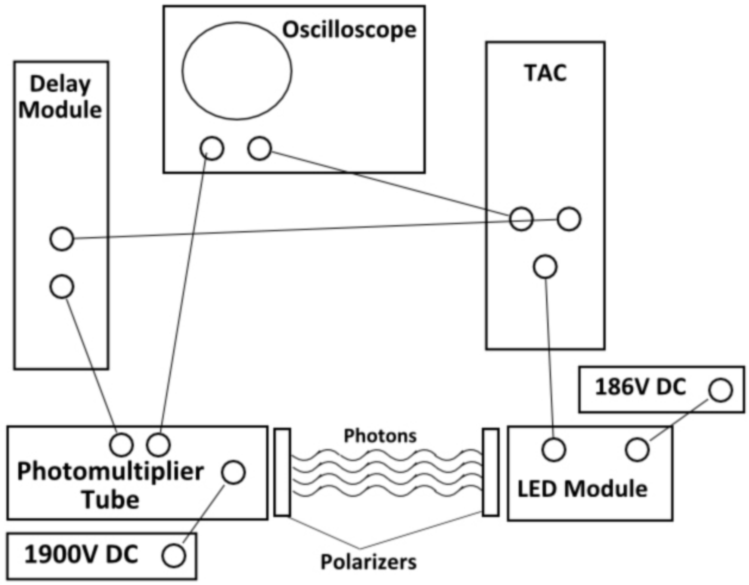

Figure 1 – *Setup diagram, with connections between instruments.*

- Various BNC wires
- 2 Polarizing filters

Setup

As described in Dr. Gold's Junior Laboratory Manual Gold (2006), and clarified in Figure 1.

- The LED module is connected to its power supply. It also has a BNC jack which has a voltage applied across it when the timing circuit turns on the LED. This BNC jack is connected to the first trigger input on the TAC. The LED module also has the meter sticks taped to it, and one of the polarizing filters is placed in the path of the LED. The module is inserted into one end of the cardboard tube so that the LED points down its length.
- The PMT is connected to its power supply. Its anode is connected to the input on the delay module and to channel 1 of the oscilloscope. It has the other polarizing filter placed in front of its collecting end. The PMT is inserted into the opposite end of the long cardboard tube, with the collecting end pointed at the LED module.
- The delay module's output is connected to the second trigger input on the TAC.
- The TAC (which now has 2 inputs connected) has its output connected to channel 2 of the oscilloscope. The output will have a voltage across it proportional to the time in between trigger events. The constant of proportionality can be set with a knob on the face of the TAC. For this report, the TAC is set to output a voltage of $\frac{1}{5}$ Volts per nanosecond.
- The PMT power supply is set to around 1900 volts DC, and the LED power supply is set to around 186 volts DC.

Procedure

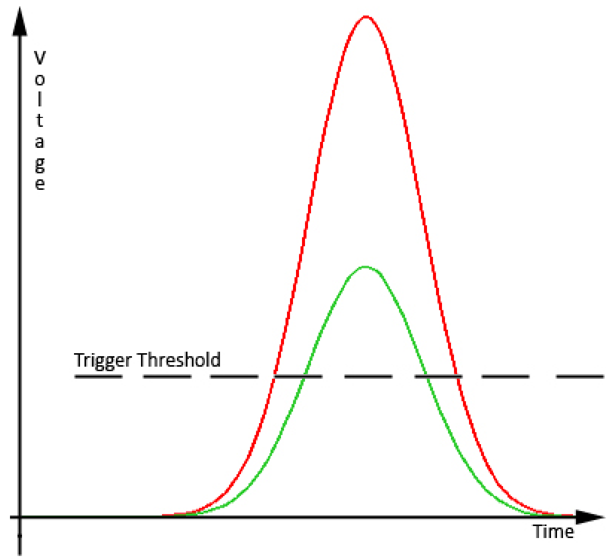

Figure 2 – *Illustration of timewalk. Note that both pulses begin and end at the same time, but the pulse with larger amplitude crosses the trigger threshold at an earlier time than the smaller amplitude pulse.*

- Turn the power supplies, TAC and DSO on. The LED module should be firing now, and the PMT should be registering a corresponding drop in potential for every pulse of incident light.
- The TAC will be triggered by the LED module pulsing. It will be triggered again by a dip in potential across the photomultiplier tube caused by incident photons striking the photocathode material on the end of the PMT and the resulting cascade of electrons moving towards the anode. The TAC then creates a potential across the two output leads which is proportional to the time between being triggered on and off. The oscilloscope measures this voltage.
- We must be careful of a very large source of systematic error: timewalk. Timewalk is an interesting phenomenon which is explained very well in Dr. Gold's manual Gold (2006), but the essence is this: the TAC is triggered at a set voltage. This voltage will be reached sooner if the pulse being sent to the TAC is larger, and later if the pulse is smaller (see Figure 2). The size of the pulse is proportional to the brightness of the incident light on the PMT, which is proportional to the distance between the LED source and the PMT detector. To control this effect, a reference voltage is taken from the PMT which indicates the brightness of the incident light. The polarizers in front of the source and emitter are turned as the distance changes in order to keep the brightness the same, indicated by the same reference voltage.
- By varying the distance between the LED module and the photomultiplier tube and taking voltage measurements, we can determine the speed of light. Plot the distance vs. time and take the slope of the line connecting these points to get a rough estimate. By finding the slope of a line fit using the least-squares method, we can get a better estimate.

3 Results and Discussion

Analysis

The speed of light is the slope of a line fit by the least squares method. The line is of the form $y = mx+b$, where m is the slope and b is the y-intercept.

According to Taylor (1997), the slope of this line is $m = \dfrac{\sum x^2 \sum y - \sum x \sum xy}{\Delta}$, and the y-intercept of the line is $b = \dfrac{N\sum xy - \sum x \sum y}{\Delta}$, where $\Delta = N\sum x^2 - (\sum x)^2$ and N is the number of data.

The standard error of the slope is $\sigma_m = \sigma_y \sqrt{\dfrac{N}{\Delta}}$ where $\sigma_y = \sqrt{\dfrac{1}{N-2}\sum_{i=1}^{N}(y_i - b - mx_i)^2}$, and the standard error of the y-intercept is $\sigma_b = \sigma_y \sqrt{\dfrac{\sum x^2}{\Delta}}$ (Taylor, 1997).

In analyzing the data (see Table 1 in Addendum) from this experiment, the measured times are the x-values, and the distances are the y-values.

Thus, our most likely slope is 3.063×10^8 meters per second, and our most likely y-intercept is -6.60 meters. The standard error of the slope is 1.83×10^7 meters per second, and the standard error of the y-intercept is 4.24×10^{-1} meters. It's worth noting that the y-intercept is nonzero, which implies that even if the emitter and detector had no distance between them the TAC would still output a voltage. This is because the electrical signals travel through the BNC connections at some finite speed, and the cables are different lengths.

The most likely slope line is produced by pairing the most likely slope and most likely y-intercept. The maximum slope is the mean slope plus the standard error of the slope and the minimum slope is the mean slope minus the standard error of the slope. The maximum and minimum y-intercepts, similarly, are the mean y-intercept plus and minus the standard error of the y-intercept, respectively.

The maximum slope line comes and pairing the maximum slope and minimum y-intercept, while the minimum slope line is the pairing of the minimum slope and maximum y-intercept. Figure 3 is a plot of the data, most likely slope line, and maximum and minimum slope lines.

Conclusions

While our result of $(3.06 \pm 0.18) \times 10^8$ meters per second is in good agreement with the accepted value of 2.998×10^8 meters per second, there was a relative uncertainty of 5.96%. If more measurements were to be taken, this relative uncertainty could shrink considerably.

A possible systematic source of error is the equipment. The reference voltage of the photomultiplier tube was inconsistent and varied from the recorded value by ± 4 millivolts to ± 8 millivolts. The cause for this inconsistency is uncertain; perhaps the LED module was not firing with a consistent voltage (and hence had a variable intensity).

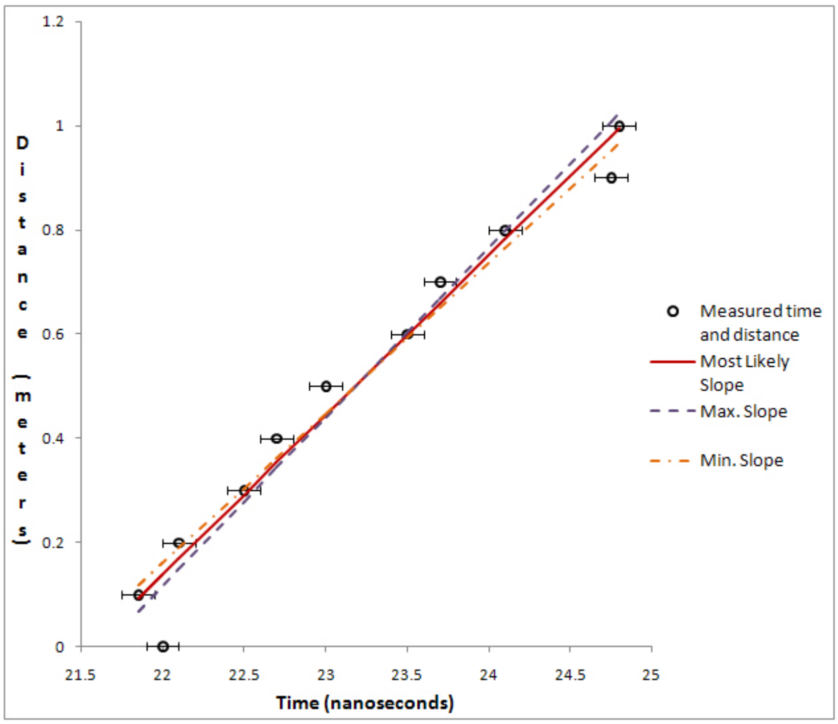

Figure 3 – *Plot of distance (in meters) from meter stick reading of 0.7m vs. measured times (in nanoseconds). The most likely slope, maximum and minimum slopes (which are one standard error higher and lower than the most likely slope, respectively) are also shown.*

References

Essen, L., & Gordon-Smith, A. C. (1948). *Proceedings of the Royal Society*.

Fizeau, H. L. (1849). Sur une exprience relative la vitesse de propogation de la lumire. *Comptes Rendus*, *29*, 90–92.

Gold, M. (2006). *PHYSICS 307L: Junior Laboratory*. The University of New Mexico, Dept. of Physics and Astronomy. <http://www-hep.phys.unm.edu/~gold/phys307L/manual.pdf> [Online; accessed 6-November-2007].

Michelson, A. A., Pease, F. G., & Pearson, F. (1935). Measurement of the velocity of light in a partial vacuum. *Astrophysical Journal*, *82*.

Taylor, J. R. (1997). *An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements*, chap. 8, (pp. 181–190). University Science Books, 2 ed.

Tholen, A. D., Barbrow, L. E., & Heffernan, A. P. (1983). Report of the 68th National Conference on Weights and Measures 1983. *NASA STI/Recon Technical Report N*, *84*, 17425–+.

Wikipedia (2007). Speed of light — wikipedia, the free encyclopedia. <http://en.wikipedia.org/w/index.php?title=Speed_of_light&oldid=169504559> [Online; accessed 6-November-2007].

4 Addendum

Data

Measured Voltages					
Meter Stick Reading (in cm)	Distance from Reading of 140cm (in m)	Voltage	Error	Time (n Sec)	Error (n Sec)
40	1.0	4.96	± 0.02V	24.80	± 0.10
50	0.9	4.95	± 0.02V	24.75	± 0.10
60	0.8	4.82	± 0.02V	24.10	± 0.10
70	0.7	4.74	± 0.02V	23.70	± 0.10
80	0.6	4.70	± 0.02V	23.50	± 0.10
90	0.5	4.60	± 0.02V	23.00	± 0.10
100	0.4	4.54	± 0.02V	22.70	± 0.10
110	0.3	4.50	± 0.02V	22.50	± 0.10
120	0.2	4.42	± 0.02V	22.10	± 0.10
130	0.1	4.37	± 0.03V	21.85	± 0.15
140	0.0	4.40	± 0.02V	22.00	± 0.10

Table 1 – *These are measurements taken from the Time-Amplitude Converter using the dual channel oscilloscope. The voltages and their errors are the result of our best judgment by watching the Channel 1 "min" reading on the oscilloscope, set to average over 128 measurements. Typically, the Channel 1 minimum reading was unstable and varied between ± 0.02V or 0.03V, and seemed to spend most of the time around the recorded mean. This is, of course, not objective and could be a source of error. The third column ("Voltage") was the output of the function "min" for Channel 1 of the oscilloscope. The corresponding times in nanoseconds are the product of the voltage and 5 because the TAC was set to $\frac{1}{5}$ Volts per nanosecond. The errors of the times are related to the errors of the voltages by the expression $\Delta time = 5 \times \Delta Voltage$, where $\Delta Voltage$ and $\Delta time$ represent the uncertainty in voltage and time.*