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BIOL 398/MATH 388 WEEK 4 ASSIGNMENT:
Two nutrients and population growth in a Chemostat

(1) $\frac{dx}{dt} = -qx + V\frac{y}{K+y}\cdot\frac{z}{M+z}\cdot x$

$= \left(-q + V\frac{y}{K+y}\frac{z}{M+z}\right)x$

If $x \neq 0$, then

$0 = \left(-q + V\frac{y}{K+y}\frac{z}{M+z}\right)$

$\frac{q}{V} = \frac{y}{K+y}\frac{z}{M+z}$

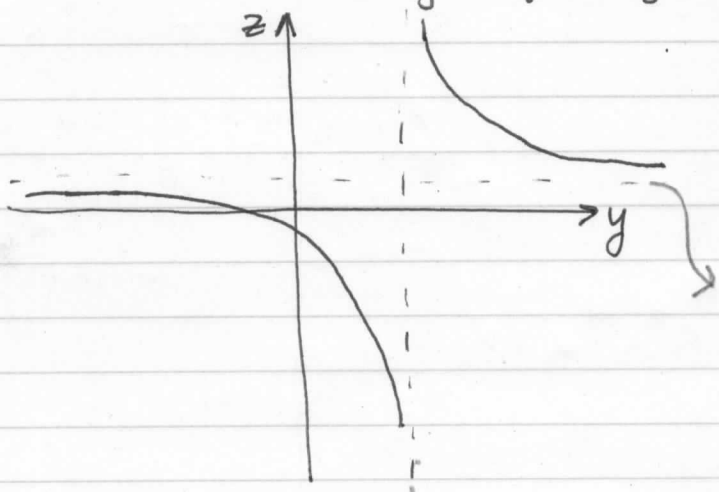
$\frac{q(K+y)}{Vy} = \frac{z}{M+z}$

$z = \frac{q(K+y)(M+z)}{Vy}$

$= \frac{qm(K+y)}{Vy} + \frac{qz(K+y)}{Vy}$

$z\left(1 - \frac{q(K+y)}{Vy}\right) = \frac{qm(K+y)}{Vy}$

$z = \frac{qm(K+y)}{Vy - q(K+y)} \Rightarrow z$ as a function of y



$qm(K+y) = 0 \Rightarrow y = -K$

$Vy - q(K+y) = 0 \Rightarrow y = \frac{qK}{V-q}$

(2) $\frac{dy}{dt} = qu - qy - \frac{1}{\varepsilon} V \frac{y}{K+y} \cdot \frac{z}{M+z} x$, where $\frac{q}{V} = \frac{y}{K+y} \cdot \frac{z}{M+z}$

$0 = q(u-y) - \frac{1}{\varepsilon} V \cdot \frac{q}{V} \cdot x$

$\frac{q}{\varepsilon} x = q(u-y) \Rightarrow x = (u-y)\varepsilon$

(3) $\frac{dz}{dt} = qv - qz - \frac{1}{\delta} V \frac{y}{K+y} \cdot \frac{z}{M+z} \cdot x$, where $\frac{q}{V} = \frac{y}{K+y} \cdot \frac{z}{M+z}$

$0 = q(v-z) - \frac{1}{\delta} V \cdot \frac{q}{V} \cdot x$

$x = \frac{q(v-z)\delta}{q} \Rightarrow x = (v-z)\delta$

$\therefore (v-z)\delta = (u-y)\varepsilon$

$-z\delta = (u-y)\varepsilon - v\delta$

$z = v - \frac{(u-y)\varepsilon}{\delta} \Rightarrow z$ as a function of y.

(3) $z = \frac{qm(K+y)}{vy - q(K+y)} = v - \frac{(u-y)\varepsilon}{\delta}$

$(qmK + qmy)\delta = (v - \varepsilon u + \varepsilon y)(vy - qK - qy)$

$qmK\delta + qmy\delta = v^2y + \varepsilon uqK - vqK - vqy - \varepsilon uvy + \varepsilon uqy + \varepsilon vy^2 - \varepsilon qKy - \varepsilon qy^2$

$qmK\delta - \varepsilon uqK + vqK = v^2y - vqy - \varepsilon uvy + \varepsilon uqy + \varepsilon vy^2 - \varepsilon qKy - \varepsilon qy^2$

$qK(m\delta - \varepsilon u + v) = y^2(\varepsilon v - \varepsilon q) + y(v^2 - vq - \varepsilon uv + \varepsilon uq - \varepsilon qK)$

$0 = \varepsilon(v-q)y^2 + (v(v-q) + \varepsilon(uq - uv - qK))y - qK(m\delta - \varepsilon u + v)$

$0 = ay^2 + by + c$, where

$a = \varepsilon(v-q)$

$b = (v(v-q) + \varepsilon((u-K)q - uv))$

$c = -qK(m\delta - \varepsilon u + v)$

$\therefore y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(3) $$y = \frac{-(v(v-q)+\varepsilon((u-K)q-uv)) \pm \sqrt{(v(v-q)+(\varepsilon((u-K)q-uv)))^2 - 4\varepsilon(v-q)(-qK(m\delta-\varepsilon u+v))}}{2\varepsilon(v-q)}$$

$$= \frac{-v(v-q)-\varepsilon((u-K)q-uv) \pm \sqrt{v(v-q)+(\varepsilon((u-K)q-uv))^2 - 4\varepsilon(v-q)(-qK(m\delta-\varepsilon u+v))}}{2\varepsilon(v-q)}$$

(4) $x = (u-y)\varepsilon$ OR $x = (v-z)\delta$

substitute y from above (3).

$$= \left(v - \frac{qm(K+y)}{vy-q(K+y)}\right)\delta$$

$$= \frac{v^2y - qv(K+y) - qm(K+y)}{vy-q(K+y)} \cdot \delta$$

$$x = \frac{v^2y - q(K+y)(v+m)}{vy-q(K+y)} \cdot \delta$$

(5) Graph of nutrients' and biomass' concentrations as a function of feed rate u of first nutrient.