20.309 / 2.673 / MAS.402
Biological Instrumentation and Measurement, Fall 2008
Department of Biological Engineering
Massachusetts Institute of Technology

Problem Set #6 Due: Tuesday, October 28

1. Light as EM waves. The propagation of light in free space can be described by electromagnetic waves as discussed in lecture. Green light has wavelength, λ, of 500 nm. Recall that the relationship between the speed of light, $c=3x10^8$ *m/s*, and its frequency, f, and wavelength is: $c = \lambda f$. For all cases, assume the wave amplitude is zero at the origin at time zero.

(a) Write an expression for a spherical wave of green light with its source at the origin.

(b) Write an expression for a plane wave of green light with its origin at the y-z plane propagating along +x direction.

(c) Write an expression for a plane wave of green light with its origin at the y-z plane propagating along -x direction.

(d) Consider the +x going plane wave, plot the wave amplitude as a function time at a distance +100 μm along x direction from the y-z plane. Plot the wave amplitude as a function of distance at t=1 sec.

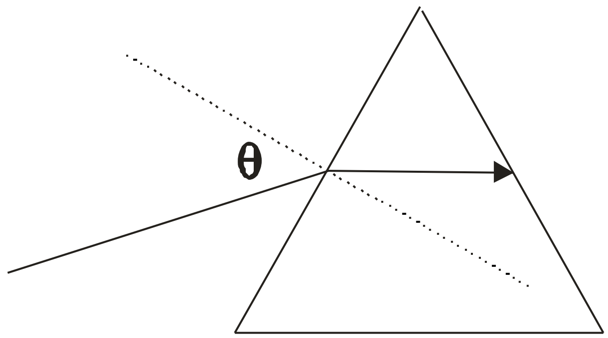
(e) The intensity (energy) of light is the temporal average of the squared amplitude of the field intensity:

$$I(r) \propto \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} E^2(r,t)dt \quad \text{where } T = \frac{1}{f} \text{ is the period of the wave.}$$

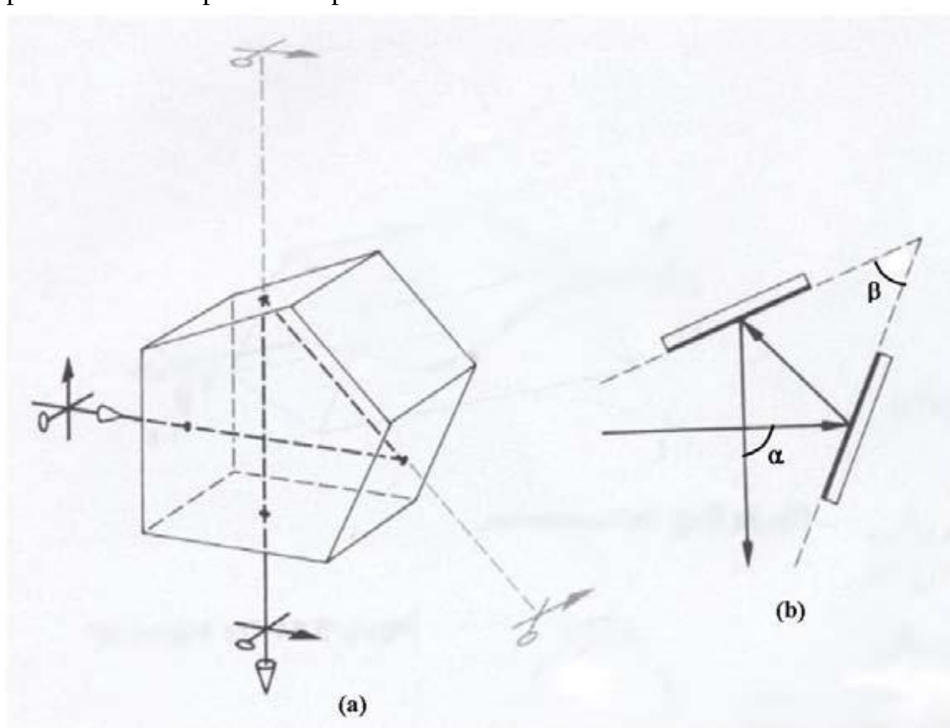
For both the plane and spherical wave cases, plot the intensity of light per unit area as a function of distance (x for plane wave and r for spherical wave) from the origin (normalizing relative to the value at the origin).

2. Prisms. Consider light propagation through prisms.

(a) For an iscoles triangle prism (all angles are 60°) made of glass (n=1.5), what is the incident angle such that the light ray inside the prism will travel parallel to one of its edge. What is the direction of the ray after it emerges from the prism?



(b) The figure below shows a penta prism and its equivalent mirror set. Show that the angle α is always equal to twice the angle β between the plane mirrors, regardless of the angle-of-incidence. Why is the penta prism a useful optical component?



3. Koehler illumination. Consider two alternative specimen illumination systems for microscopes (a) & (b).

In configuration a, the light from the filament is projected to the sample plane using two lenses. The filament locates at f1, the front focal plane of the left lens, and the sample plane locates at f2, the back focal plane of the right lens. The focal points of the two lenses coincide on the optical axis. In configuration b, the light from the filament is projected to the sample using one lens. The filament sits at the front focal plane while the sample plane sits at the back focal plane. By ray tracing and analytical arguments, what are the light distributions at the focal plane? Which one is the more appropriate configuration and why?

4. Interference of plane waves. Consider two plane waves with the same wavelength, λ, but different propagation directions:

$$E_1(y,z,t) = E_0(\vec{k}_1 \cdot \vec{r} - \omega t)$$

$$E_2(y,z,t) = E_0(\vec{k}_2 \cdot \vec{r} - \omega t)$$

where $\vec{k}_1 = \frac{2\pi}{\lambda}(\sin\theta\,\hat{z} + \cos\theta\,\hat{y})$ and $\vec{k}_2 = \frac{2\pi}{\lambda}(\sin\theta\,\hat{z} - \cos\theta\,\hat{y})$.

What are the light intensity distributions due to interference for θ =0°, 30°, 45°, 60°, 90°? Plot these distributions.

5. Images from lens combinations. A lens combination consists of two lenses, L1, and L2, with focal lens of 100 and 200 mm respectively. The object to be imaged is an up-right arrow with a size of 1 mm placed at a distance of 150 mm in front of L1. Describe the image formed by the lens combination for three different lens separation distances, x, of 400, 600, and 800 mm.