$$\dot{C} = -\mu(c)(M-s) + u(s_{in} - s)$$

$$\dot{M} = u(s_{in}$$

$$\dot{C} = -\mu(c)(M-c) + \bar{u}(c_{in} - c)$$

$$\dot{M} = u(c_{in} - c - \alpha(M-c))$$

$$\dot{X} = \mu(c)X - Dx$$

$$\dot{C} = -\mu(c)X + D(c_{in} - c)$$

D : dilution rate
C : ammonia concentration
X : biomass
μ : growth function
$C_{in} > 0$: input ammonia concentration

$$H(c, X, \lambda_c, \lambda_X, \lambda_0, u) = \cdots$$

$$H = -\lambda_c \mu(c) X + \lambda_0 +$$

$\dot{C} = -\mu(c)(M-c) + u(c_{in}-c)$

$\dot{M} = u(c_{in} - c - \alpha(M-c))$

$M = x + 6$; total mass of system

$(6_u(\cdot), M_u(\cdot))$ unique solution defined over $[0, \infty)$ associated to control $u \in \mathcal{U}$ s.t. $C_u(0) = C_0$ and $M_u(0) = x_0 + C_0$ at $t = 0$.
$\bar{M} = \bar{x} + \bar{6}$

given target point $(\bar{x}, \bar{c})$ we aim to steer equation (1) in minimal time from $(x_0, c_0) \to (\bar{x}, \bar{c})$

$H = -\lambda_c \mu(c)(M-6) + \lambda_0 + u[(\lambda_c + \lambda_m)(c_{in}-c) - \alpha\lambda_m(M-c$

$$\frac{\mu c}{K+c}$$

$\lambda_m(c_{in} - M) + \lambda_c(c_{in} - c)$

$$\frac{(K+c)\mu - \mu c(K)}{(K+c)^2}$$

$$\mu K + \mu c - \mu c K$$

$$\frac{\mu(c + K - cK)}{(K+c)^2}$$

$$\frac{dx_1}{dt} = \left(\frac{0.4\,x_2}{x_2+133}\right)x_1 - 0.1\,x_1$$

$$\frac{dx_2}{dt} = 0.1\,(x_2 - u) - \left(\frac{0.4\,x_2}{x_2+133}\right)x_1$$

$$\frac{dx_3}{dt} = \frac{u}{500}$$

$$0 \le x_1 \le 40$$

$$0 \le x_2 \le 40$$

$$0 \le x_3 \le 10$$

$$0 \le u \le 50$$

$u = 0.4$

$$\frac{dX}{dt} = \mu X - \frac{F}{V} X = \left(\frac{\mu_0}{1+P/K_P}\right)\left(\frac{S}{K_S+S}\right) X - \frac{F}{V} X$$

$$\frac{dS}{dt} = -\frac{\mu}{Y_{X/S}} X + \frac{F}{V}(S_F - S)$$

$$\frac{dP}{dt} = \pi X - \frac{F}{V} P$$

$$\frac{dV}{dt} = F$$

$$\mu = \frac{\mu_0}{1+P/K_P} \frac{S}{K_S+S} \quad , \quad \pi = \frac{\pi_0}{1+P/K_P} \frac{S}{K_S+S}$$

X: cell mass conc.
S: substrate concentration
P: product concentration
μ: specific growth rate
π: specific productivity
F: feed rate

$S_F = 150\,g/L$
$S_0 = 150$
$V_0 = 10$
$t_f = 54h$

0 0

$$J = P(t_f)\,V(t_f)$$