UNIVERSIDAD DE MÁLAGA

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Control Theory

Introduction to Synthetic Biology

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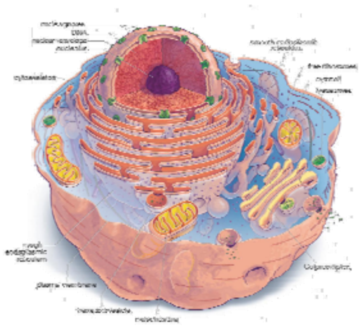
InterTech

Overview

- Introduction
- Types of controllers
- Block diagram
- Example of control
- Laplace transform and transfer function
- Frequency domain
- Examples

- It consist of “controlling”.

Maintain between certain levels the value of something



Krogh, *Biology*, Custom Core Edition, **Prentice Hall**, 3rd ed., 2004

- It is a wide branch of knowledge that has been studied since the beginning of times.
- Kybernetes, Greek word for navigator, steers-man, related to the Latin gubernator (governor). In "The Republic", by Plato (428-347 BC) steering a ship was compared to steering a community. Aristotle used kybernetike to refer to steering a community.
- It could be even related to AI.

Control loop examples

Open loop

Closed loop

Open loop

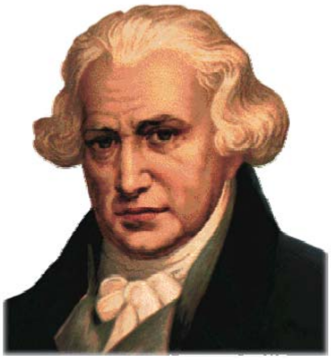
Development of clocks

Huygens

Hooke

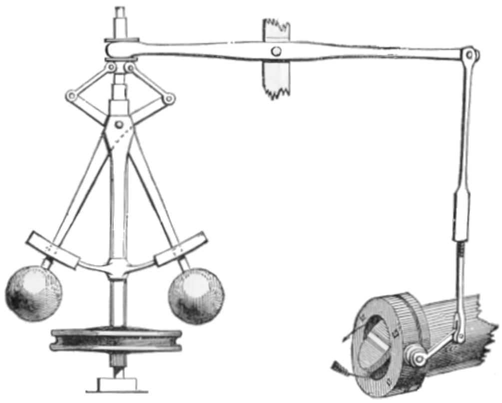
Closed loop

Speed regulation in the first steam engines



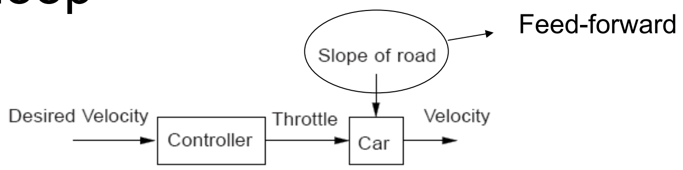
James Watt

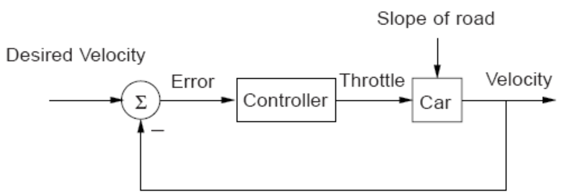
Matthew Boulton



Central governor

A clear example of closed and open loop





Closed loop

- Reduces effects of process disturbances
- Makes the system insensitive to process variations
- Stabilize an unstable system
- Create well-defined relations between output and reference
- Risk for instability

Open loop

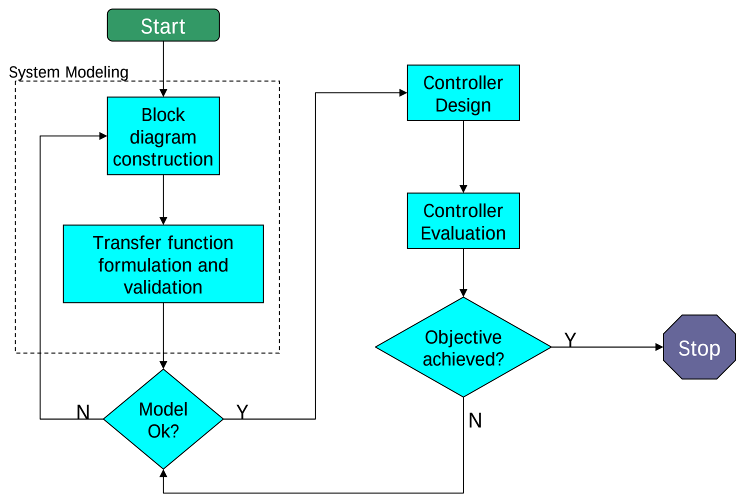
- Reduces effects of disturbances that can be measured
- Improve response to reference signals
- No risk for instability
- Design of feed-forward is simple but it requires good models
- Beneficial to combine with feedback

Lecture by Wilbur Wright 1901:

> *Men know how to construct airplanes.*
> *Men also know how to build engines.*
> *Inability to balance and steer still confronts students of the flying problem.*

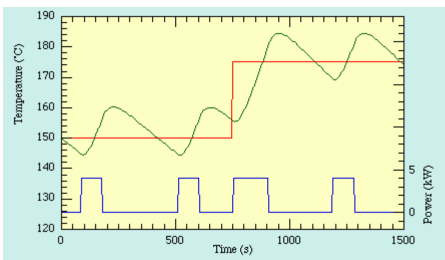
When this feature has been worked out, the age of flying arrived, because all other difficulties were of minor importance.
The Wright Brothers figured it out and flew the Kitty Hawk on December 17 1903!

Controller Design Methodology



On/off control loop

- The simplest control.
- It is used for almost all the thermostats.



P-controller

Proportional action:
•Adjustable gain (amplifier)

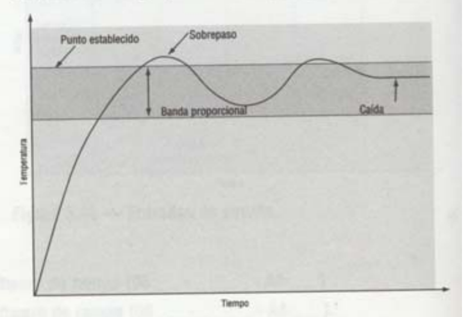


Figura 5.6b — Controlador proporcional.

u= ke

PI-controller

Integral Action:

- Eliminates bias steady-state error)
- Can cause oscillations

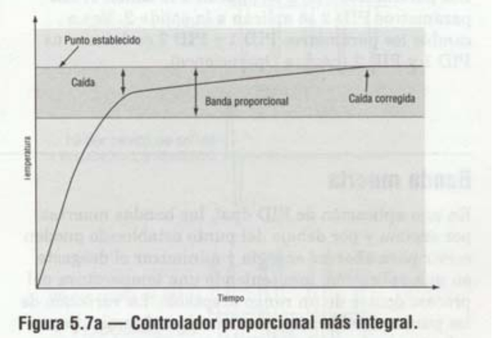


Figura 5.7a — Controlador proporcional más integral.

$$u = ke + k_i \int_0^t e(\tau)d\tau$$

An important property of PI

Consider a PI controller:

$$u = ke + k_i \int_0^t e(\tau)d\tau \longrightarrow u = ke_0 + k_i e_0 t$$

Assume that there is an equilibrium under constant $e(t)=e_0$ and constant $u(t)=u_0$. Then we must have $e_0=0$.

A PI controller allows a perfect control of the system

Internal Model Principle (IMP)

- Internal Model Principle is a generalization of the necessity of an integral control.
- Robust tracking of an arbitrary signal requires a model of that signal, in the controller.
- By intuition, the internal model counteracts the external signal.

PID

- Derivative Action ("rate control"):
  - Effective in transient periods
  - Provides faster response (higher sensitivity)
  - Never used alone

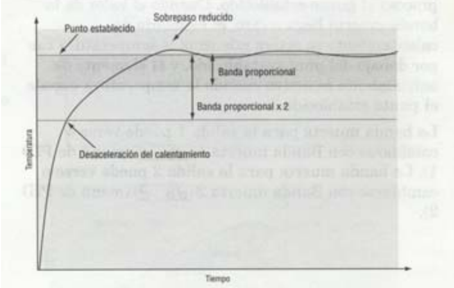
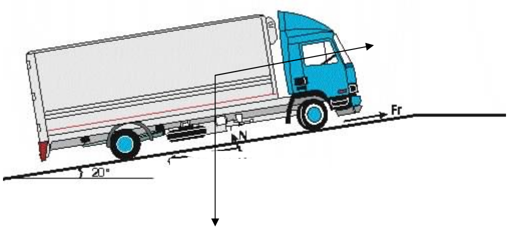


Figura 5.7b — Controlador PID.

$$u = ke + k_i \int_0^t e(\tau)d\tau + k_d \frac{de(\tau)}{d\tau}$$

Example of control:Cruise control

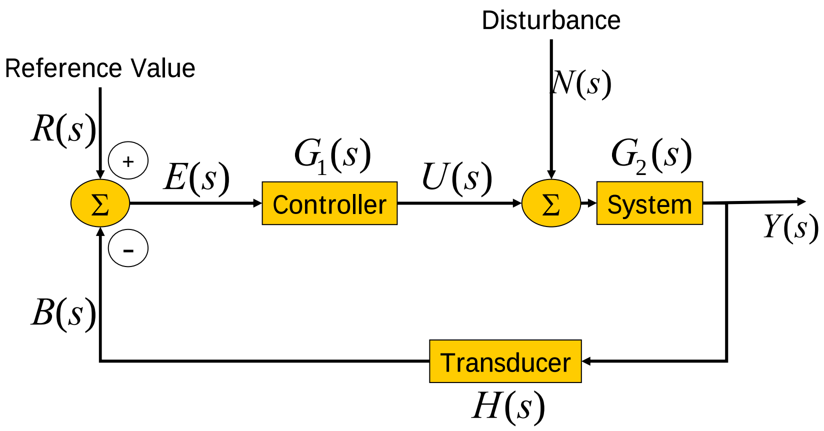


- Control variable: gas pedal (throttle) u
- Process output: velocity v
- Desired output or reference signal v_r
- Disturbances: slope θ

Block Diagrams

- Pictorially expresses flows and relationships between elements in the system
- Blocks may recursively be systems
- Rules
  - Cascaded (non-loading) elements: convolution
  - Summation and difference elements
- Can simplify the evaluation

Block Diagram of System

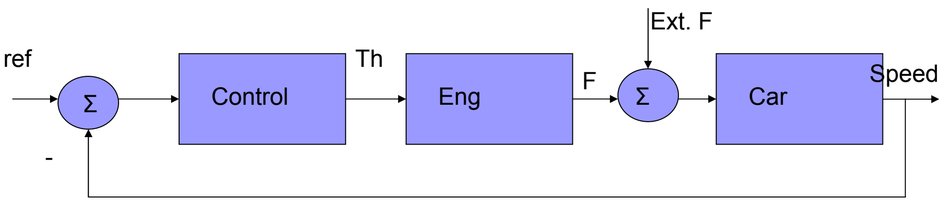


Example of control: Cruise control

The block diagram gives an overview.

To draw a block diagram:

- Understand how the system works.
- Identify the major components and the relevant signals.
- Key questions:
  - Where is the essential dynamics?
  - What are the appropriate abstractions?
- Describe the dynamics of the blocks.



Simplifying assumptions

· Essential dynamics relates velocity and force
· The force responds very fast to a change in the throttle
· Assume that all relations are linear (small perturbations)

Modelling the system

$$m\frac{dv}{dt} \quad vd \quad F \quad mg \xrightarrow[\text{value for the constants}]{\text{With reasonable}} \frac{dv}{dt} \quad 0.02v \quad u \quad 10$$

Inserting a PI controller $\longrightarrow u \quad k(v_r \quad v) \quad k_i \int_0^t (v_r \quad v(\ ))d$

The closed loop velocity control device is described by:

$$\frac{d^2e}{dt^2} + (0.02 + k)\frac{de}{dt} + k_i e = 10\frac{d\theta}{dt}$$

In steady state (a constant slope) e=0 $\longrightarrow$ PI Control!!

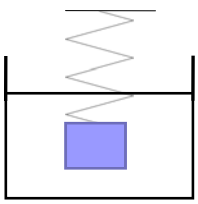
Small Problem

What is the influence of the PI parameters?

$$\frac{d^2e}{dt^2} \quad (0.02 \quad k)\frac{de}{dt} \quad k_i e \quad 0 \longrightarrow \frac{d^2x}{dt^2} \quad 2 \quad {}_0\frac{dx}{dt} \quad {}_0^2x \quad 0$$

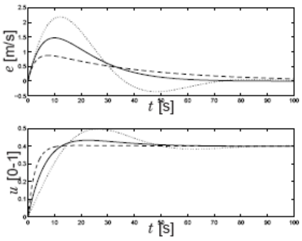
Spring mass damper

ω : Gives the response speed
Σ : The shape of the response

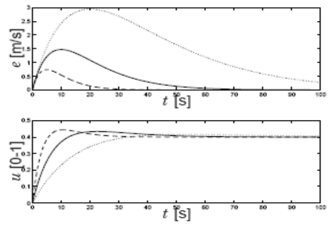


$$k_i = \omega_0; k = 2\varsigma\omega_0$$

$\omega = 0.1$, $\zeta = 0.5$ (dotted), $\zeta = 1$ (solid), and $\zeta = 2$ (dashed)



$\zeta = 1$, $\omega_0 = 0.05$ (dotted), $\omega_0 = 0.1$ (solid) and $\omega_0 = 0.2$ (dashed)



http://www.control.lth.se/~kja/

Linearized equations

Formulate a liniarized model following this form:

$$\frac{d^n y}{dt^n} + a_1 \frac{d^{n-1} y}{dt^{n-1}} + \dots + a_n y = b_1 \frac{d^{n-1} u}{dt^{n-1}} + \dots + b_n u$$

$$\frac{dy}{dt} + a_n y = 0$$

$$y = Ce^{-a_n t}$$

$$y = \sum_{k=1}^{n} C_k e^{-a_k t}$$

$$\frac{dy}{dt} + a_n y = b_n u$$

$$y = Ce^{-a_n t} + b_n \int_0^t e^{a_n (t-\tau)} u(\tau) d\tau$$

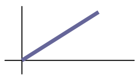
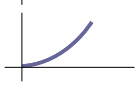
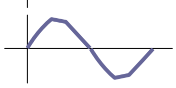
$$y = \sum_{k=1}^{n} C_k e^{-a_k t} + \int_0^t g(t-\tau) u(\tau) d\tau$$

Basic Tool For Continuous Time: Laplace Transform

$$\mathbf{L}[f(t)] = F(s) = \int_0^\infty f(t)e^{-st}dt$$

- Convert time-domain functions and operations into frequency-domain
  - $f(t) \rightarrow F(s)$
  - Linear differential equations (LDE) $\rightarrow$ algebraic expression in Complex plane
- Graphical solution for key LDE characteristics
- Discrete systems use the analogous z-transform

Laplace Transforms of Common Functions

Name	$f(t)$		$F(s)$
Impulse	$f(t) = \begin{cases} 1 & t = 0 \\ 0 & t > 0 \end{cases}$		1
Step	$f(t) = 1$		$\frac{1}{s}$
Ramp	$f(t) = t$		$\frac{1}{s^2}$
Exponential	$f(t) = e^{at}$		$\frac{1}{s-a}$
Sine	$f(t) = \sin(\omega t)$		$\frac{1}{\omega^2 + s^2}$

Laplace Transform Properties

Addition/Scaling	$L[af_1(t) \pm bf_2(t)] = aF_1(s) \pm bF_2(s)$
Differentiation	$L\left[\frac{d}{dt}f(t)\right] = sF(s) - f(0\pm)$
Integration	$L\left[\int f(t)dt\right] = \frac{F(s)}{s} + \frac{1}{s}\left[\int f(t)dt\right]_{t=0\pm}$
Convolution	$\int_0^t f_1(t-\tau)f_2(\tau)d\tau = F_1(s)F_2(s)$
Initial-value theorem	$f(0+) = \lim_{s\to\infty} sF(s)$
Final-value theorem	$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$

Insights from Laplace Transforms

- What the Laplace Transform says about f(t)
  - Value of f(0)
    - Initial value theorem
  - Does f(t) converge to a finite value?
    - Poles of F(s)
  - Does f(t) oscillate?
    - Poles of F(s)
  - Value of f(t) at steady state (if it converges)
    - Limiting value of F(s) as s->0

Poles and Zeros of a Complex Function

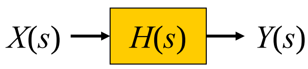
- A rational complex function f(z) can be written as

$$G(s) \quad \frac{a_0 \quad a_1 s \quad \cdots \quad a_n s^n}{b_0 \quad b_1 s \quad \cdots \quad b_m s^m}$$

- The zeros are values for which f(z) = 0.
- The poles are values for which f(z) is undefined.
- Together, the zeros and poles nearly provide a complete description of f(z).

- The differentiation property makes the Laplace transform very convenient for dealing with LTI systems, particularly if all initial values are zero. Differentiation of the time functions simply corresponds to multiplication of the transform with *s*. Then, we obtain the following recipe for linear systems:
- Take Laplace transforms of the equations
- Take Laplace transforms of the signals acting on the system
- Solve linear algebraic equations to obtain the transforms the interesting signals
- Convert the Laplace transform into a time function

Transfer Function



- Definition
  - G(s) = Y(s) / X(s)
- Relates the output of a linear system (or component) to its input
- Describes how a linear system responds to an impulse
- All linear operations allowed
  - Scaling, addition, multiplication

Model transfer function

$$m\frac{d^2y}{dt} = ku \rightarrow ms^2Y(s) = kU(S)$$

$$G(S) = \frac{Y(S)}{U(S)} = \frac{k}{ms^2}$$

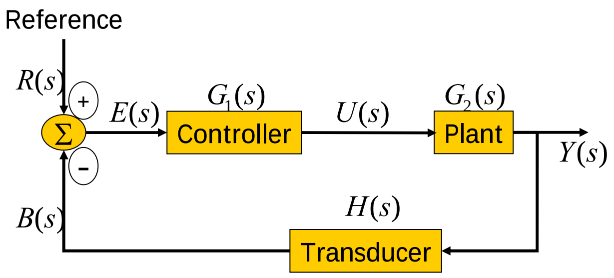
Basic Control Actions: u(t)

Proportional control: $u(t) = K_p e(t)$ $\quad \frac{U(s)}{E(s)} = K_p$

Integral control: $u(t) = K_i \int_0^t e(t)dt$ $\quad \frac{U(s)}{E(s)} = \frac{K_i}{s}$

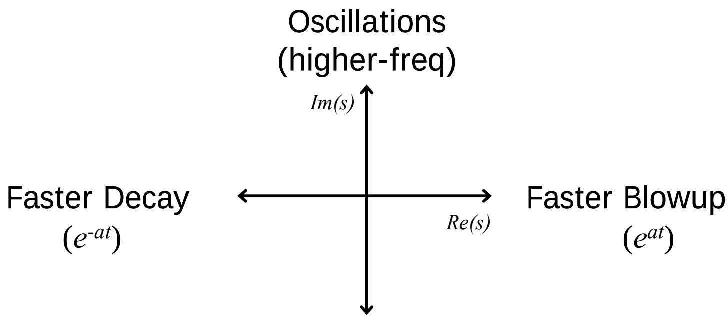
Differential control: $u(t) = K_d \frac{d}{dt} e(t)$ $\quad \frac{U(s)}{E(s)} = K_d s$

Key Transfer Functions



Feedback : $\frac{Y(s)}{R(s)} = \frac{G_1(s)G_2(s)}{1 + G_1(s)G_2(s)H(s)}$

Effect of pole locations



Why to develop this analysis?

- In principle, time responses can be computed but…
- Making order of magnitude is always a good rule
- Whenever you use software, make sure that results are reasonable.
- Much insight can be obtained from very simple calculations (series expansions and factorization).

Example

- Consider a signal given by:

$$G(s) = \frac{s+3}{(s+2)(s+4)}$$

The system will decay after a short time as the time function, associated to G(s), has two negative exponentials.

$$e^{-2t}, e^{-4t}$$

Insight from the transfer function

- Derive transfer function $G(s) = \frac{B(s)}{A(s)}$
- Compute poles α_k (roots of *A(s)*)
  - Free motion of system has component $Ce^{\alpha t}$
- Compute zeros β_k (roots of *B(s)*=0)
  - The system blocks transmission of the signal $Ce^{\beta t}$
- Compute static gain *G(*0)
- Look at behavior for small *s* (large *t*, low frequencies) and large *s* (small *t*, high frequencies)

Scheme of a control strategy

- Transform the physical problem into a standard model
- Pick a controller e.g. PI
- Design a controller where the closed loop characteristic equation has specified poles (pole placement)
- Translate results back to the physical system

- Solve an abstract problem and you will get the solution to many concrete problems.
- Solve a specific problem and you will have the solution just to that problem.

Controllability and observability

- Controllability: Hability of a system to be forced to a certain state with the proper control system. (It is not a synomimous of stabliblity).
- Observability:hability to observe the state of the system.

Observability and Detectability

$$\dot{x} = f(x, u) \qquad y = h(x)$$

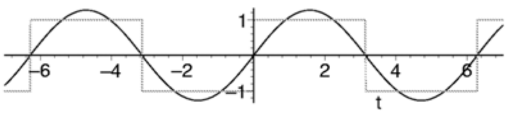
- System is observable if the state trajectory x(t) can be determined from the output trajectory y(t)
- System is detectable if: output tending to equilibrium implies state tending to equilibrium

Frequency domain

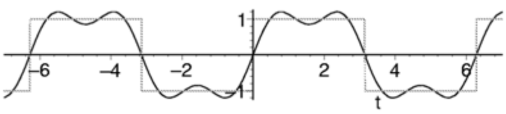
- A LTI system is completely determined by its response to sinusoidal signals.

Frequency Domain: Fourier Series

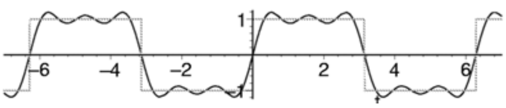
$\frac{4}{\pi}\sin(t)$



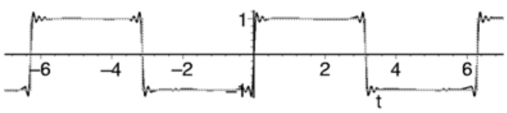
$\frac{4}{\pi}\left(\sin(t) + \frac{\sin(3t)}{3}\right)$



$\frac{4}{\pi}\left(\sin(t) + \frac{\sin(3t)}{3} + \frac{\sin(5t)}{5}\right)$

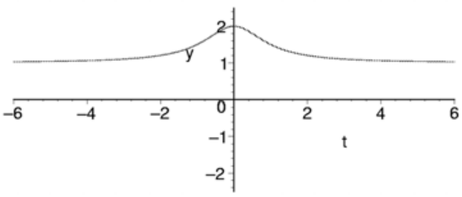


$\frac{4}{\pi}\sum_{n=1}^{20} \frac{\sin(2n-1)t}{2n-1}$

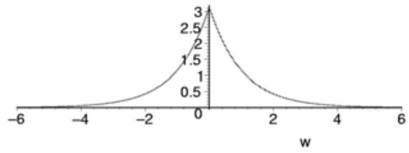


Frequency Domain: Fourier Transform

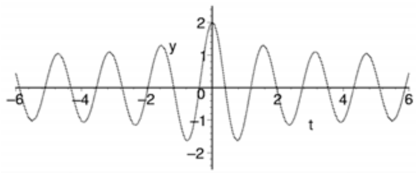
$f(t) = \frac{t^2+2}{t^2+1}$



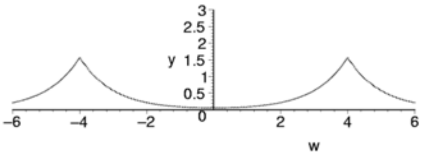
$F(i\omega)$



$g(t) = \cos(4t)\frac{t^2+2}{t^2+1}$



$G(i\omega)$

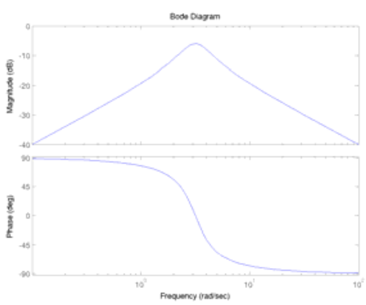


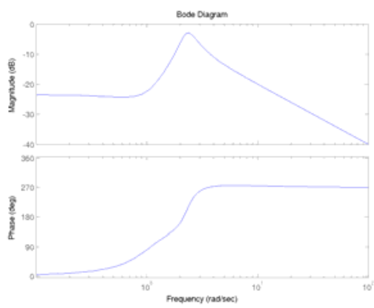
Frequency Response: Bode Plots

Superposition: a plot of system response versus frequency completely, characterizes the system

$$H(i\omega) = \frac{i\omega}{-\omega^2 + 2i\omega + 10}$$

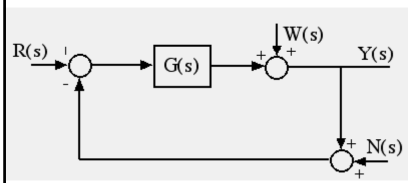
$$H(i\omega) = \frac{-\omega^2 + i\omega + 1}{-i\omega^3 - 2\omega^2 + 3i\omega + 15}$$





Bode's Sensitivity Integral

- S(ω) is the sensitivity function, it is represented by the ratio between the output signal and the system response
- Bode and others showed for *linear* systems $$\int_0^{\infty} \log|S(\omega)|\ d\omega \geq 0$$
- Termed Waterbed effect.
- Doyle and colleagues have extended Bode's formula to nonlinear systems and provided a more general interpretation.



R(s)=Y(s) Perfect tracking

Y(s)=T(s)[R(s)-N(s)]-S(s)W(s)

The typical situation is that *R, W* are small for large frequencies, and *N* for small frequencies .

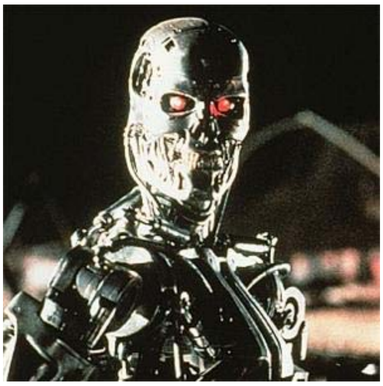
- |S(jω)| be small for small ω , that means the effect of the disturbance input is attenuated.
- |T(jω)| be small for large ω , that means the effect of the sensor noise is attenuated.
- |T(jω)| be unity (0 db) for small ω, that means the (low-frequency) characteristics of the reference input are unaffected.

Issues for Computer Science

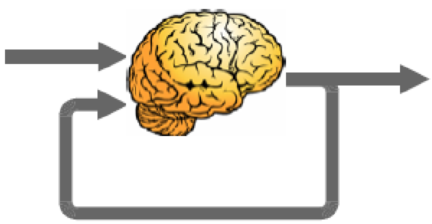
- Most systems are non-linear
  - But linear approximations may do
    - eg, fluid approximations
- First-principles modelling is difficult
  - Use empirical techniques
- Control objectives are different
  - Optimization rather than regulation
- Multiple Controls
  - State-space techniques
  - Advanced non-linear techniques (eg, NNs)

Advanced Control

- Robust Control
  - Can the system tolerate noise?
- Adaptive Control
  - Controller changes over time (adapts)
- MIMO Control
  - Multiple inputs and/or outputs
- Stochastic Control
  - Controller minimizes variance
- Optimal Control
  - Controller minimizes cost function of error and control energy
- Nonlinear systems
  - Neuro-fuzzy control
  - Challenging to derive analytic results



Aplications in Biology

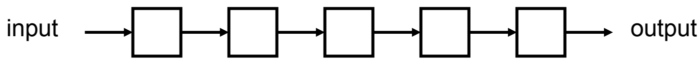


Homeostasis is Fundamental to Life

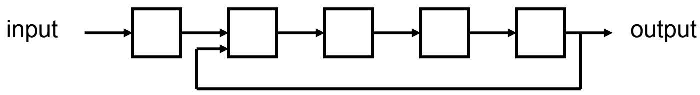
- Homeostasis is dynamic self-regulation.
- Examples: temperature, energy, key metabolites, blood pressure, immune response, hormone balance, neural functioning, etc.
- Sensory adaptation is a type of homeostasis.
- It depends on robust control.

Living Systems are Closed Loop

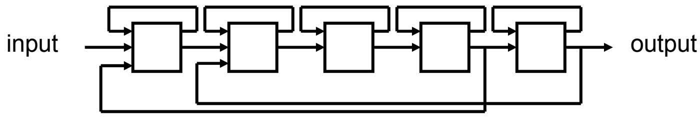
Living systems to the Naïve Biologist



Living systems to the Systems Biologist



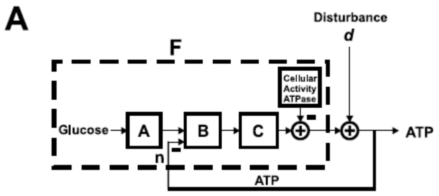
Living systems to the Enlightened Systems Biologist

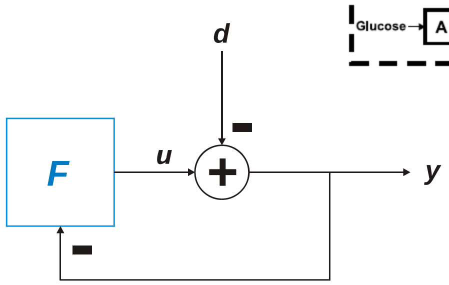


Nonlinear Control Theory is not as Well-Developed

- Many advanced techniques in control theory are only for linear systems.
- One can linearize a nonlinear system.
- Control theorists are trying to develop nonlinear analogs for linear concepts.
- Ideas from linear control provide valuable intuition.

Feedback Control and Disturbance Attenuation





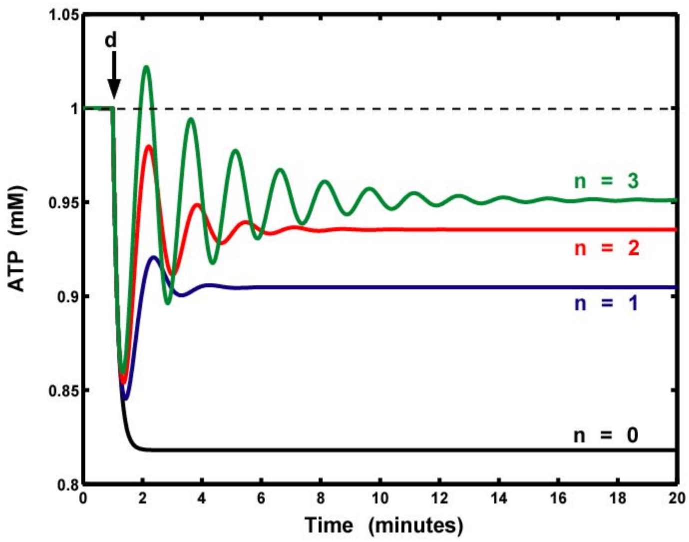
$$y = d + u = d - yF$$

$$\Rightarrow y = \frac{d}{1+F}$$

$$S = \frac{y}{d} = \frac{1}{1+F}$$

Steady-state analysis

Basic Model: Simulations



Implications of this example

- Some biological oscillations may be a side-effect of noise amplification produced by intense regulation.
- Glycolytic oscillations may arise from the tight regulation of ATP levels by the glycolytic pathway.
- Oscillations in biochemical reaction networks are a property of the whole system, not a single enzyme.
- One would expect that the tightliest regulated molecules in the cell (e.g., ATP, calcium, cAMP) would be the most susceptible to oscillations.

Bibliography

This presentation has been based on the courses:

- Lecture notes http://www.control.lth.se/~kja/ of professor KJ. Aström.
- Basic control theory for biologist. http://www.cds.caltech.edu/~tmy/ICSB2002_Tutorial, Tau-Mu Yi, Herbert Sauro and Brian Ingalls
- An Introduction to Control Theory With Applications to Computer Science of Joseph Hellerstein Sujay and Parekh. IBM T.J. Watson Research Center.

  Other interesting references for a general overview of control theory:
- Modern control theory. K Ogata. Prentice Hall.
- Optimal control theory. D Kirk. Dover Publications.

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