Question 1

$$\frac{dn}{dt} = qu_n - qn - yV\left(\frac{n}{K_n+n}\right)\left(\frac{c}{K_c+c}\right) \quad (1)$$

$$\frac{dc}{dt} = qu_c - qc - yV\left(\frac{n}{K_n+n}\right)\left(\frac{c}{K_c+c}\right) \quad (2)$$

$$\frac{dy}{dt} = ryV\left(\frac{n}{K_n+n}\right)\left(\frac{c}{K_c+c}\right) - qy \quad (3)$$

Set dn/dt, dc/dt and dy/dt to zero and solve.

Solve (3) first.

$$y\left(rV\left(\frac{n}{K_n+n}\right)\left(\frac{c}{K_c+c}\right) - q\right) = 0$$

$y = 0$ or

$$rV\left(\frac{n}{K_n+n}\right)\left(\frac{c}{K_c+c}\right) = q$$

$$\frac{rVnc}{K_nK_c+nK_c+cK_n+cn} = q$$

$$rVnc = qK_nK_c + qnK_c + qcK_n + qcn$$

$$rVnc - qnK_c - qcn = qK_nK_c + qcK_n$$

$$n(rVc - qK_c - qc) = qK_nK_c + qcK_n$$

$$n = \frac{qK_nK_c+qcK_n}{rVc-qK_c-qc}$$

I am not sure this is correct. I went on to try and solve the other two and it got messy.

Question 3

$$\frac{dn}{dt} = qu_n - qn - yV\left(\frac{n+c}{K+n+c}\right) \quad (1)$$

$$\frac{dc}{dt} = qu_c - qc - yV\left(\frac{n+c}{K+n+c}\right) \quad (2)$$

$$\frac{dy}{dt} = ryV\left(\frac{n+c}{K+n+c}\right) \quad (3)$$

Set dn/dt, dc/dt and dy/dt to zero and solve.

Solve (3) first.

$$y\left(rV\left(\frac{n+c}{K+n+c}\right)\right) = 0$$

$y = 0$ or

$\frac{rVn+rVc}{K+n+c} = 0 \rightarrow \frac{rVn}{K+n+c} = -\frac{rVc}{K+n+c} \rightarrow n = -c$

Now solve (1).

$$qu_n - qn - yV\left(\frac{n+c}{K+n+c}\right) = 0$$

From above, n = -c. Substitute below.

$$qu_n - q(-c) - yV\left(\frac{-c+c}{K-c+c}\right) = 0$$

$cq = -qu_n \rightarrow c = -u_n$

Now solve (2).

$$qu_c - qc - yV\left(\frac{n+c}{K+n+c}\right) = 0$$

Substitute.

$$qu_c - q(-u_n) - yV\left(\frac{-c+c}{K-c+c}\right) = 0$$

$qu_c + qu_n = 0 \rightarrow u_c = -u_n$

So at equilibrium when

1. $y = 0$
2. $n = -c$
3. $c = -u_n$
4. $u_c = -u_n$