IMPERIAL COLLEGE LONDON
UNIVERSITY OF LONDON

B.Eng., M.Eng and ACGI Examinations 2007

Part 3 and Part 4

Biomedical Engineering

BE3-M37/BE4-M37 Advanced Biological Modelling

Wednesday 16th May 2007 2.00pm-4.30pm

Marks are shown next to each question.
Please write answers to each Section in separate answer books.

Section A

Give brief answers to the following questions.

Total marks for this section: 20

Question 1

a) Draw a sketch of the structure of a simple perceptron with two input units, two hidden units and one output unit.

b) Assuming that each unit of the perceptron applies a nonlinear transformation g to its input, write the output y as a function of the inputs x_i , the weights of the first layer $W_{ij}^{(I)}$, the weights of the second layer $W_{ij}^{(O)}$, and the function g.

Marks: 10

Question 2

You are given $N \gg 2$ data pairs (x_i, y_i), which correspond to the measurement of a variable y as a function of a control variable x. You have strong theoretical indications that y should depend linearly on x and you need to calculate the linear coefficients of this dependency.

a) Use a sketch to explain briefly the least squares solution to this problem. In what sense is this solution optimal?

b) Write down the matrix formulation of this problem and show how to obtain the least squares solution.

Marks: 10

Section B

Answer the following two questions.

Total marks for this section: 30

Question 3

You are managing a farm that has 45 Ha in surface and you want to split your cultivated surface between wheat (W) and corn (C). The amount of labor that you can use is limited at 100 workers. Each Ha of wheat requires 3 workers while each Ha of corn requires 2 workers. The amount of fertilizer needed is 20 kg per Ha of wheat and 40 kg per Ha of corn. You can only use up to 1200 kg of fertilizer. When you go to the market, for each Ha of wheat you will get a profit of 200 while each Ha of corn returns 300.

a) Pose the optimization problem and explain how you can solve it graphically. What is the maximum profit you can get and what will be the proportion of wheat and corn cultivated?

b) Find the relative values of wheat and corn at which it will be advantageous to cultivate:
i) wheat alone;
ii) corn alone.

Marks: 15

Question 4

Consider that the population x_t of a fishery you are managing evolves in discrete time according to the difference equation

$$x_{t+1} = \frac{x_t^2 + b\,x_t}{x_t^2 + x_t + c}, \tag{1}$$

where b and c are positive parameters you can control.

a) Draw the recurrence plot for this system and find its fixed points for the different values of b and c. Find the stability of the fixed points using the cobweb analysis, or otherwise.

b) Suppose that your fishery is exposed to an epidemic that kills a fraction K of the population at each time step. Calculate the value of K that will drive your population to extinction for any given value of b and c.

Marks: 15

Section C

Answer **one** of the following two questions.

Total marks for this section: 20

Question 5

a) Define *combinatorial optimization*. Give one example of such a problem and explain what makes it usually a computationally hard problem.

b) Two methods used in combinatorial optimization are *evolutionary (genetic) algorithms* and *simulated annealing*. Describe the origin and basic ideas behind these algorithms and compare their advantages and disadvantages.

c) *For both algorithms*, write a few lines of pseudo-code to apply them to the optimization of a function $U(\chi)$, where χ is a configuration of the discrete set $\mathbf{X} = \{\chi\}$.

Marks: 20

Question 6

a) Use a sketch to explain the mathematical basis of *gradient methods* for the optimization of a function $U(x_1, x_2, \ldots, x_n)$.

b) You need to obtain numerically the *global minimum* of the function $U(x)$ in Figure 1 for which you have the explicit analytical form. Compare in detail how you would achieve it:

i) by using an integrator such as `ode45`

ii) by using the steepest descent solver `fsolve`.

c) Use this example to discuss the problems encountered when applying gradient methods to global optimization of *non-convex functions*. Explain why this is a computationally hard problem in general and mention briefly some of the methods that can help with it.

Marks: 20

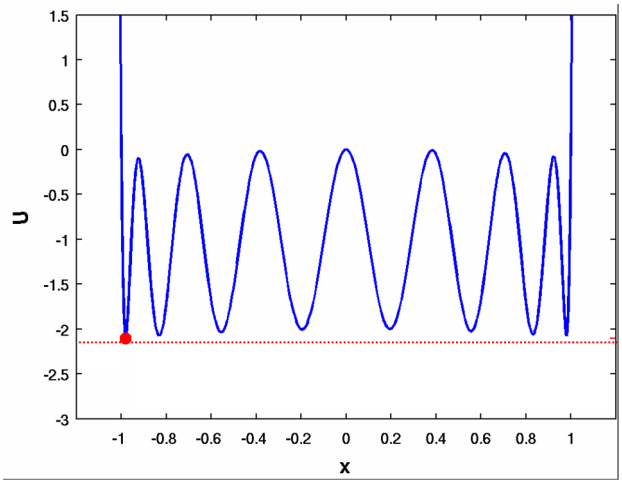

Figure 1:

Section D

Answer **one** of the following questions.

Total marks for this section: 30

Question 7

a) Use Figure 2 to explain the small-world construction.

b) Use Figure 3, extracted from the Watts-Strogatz paper, to explain the meaning of small-world in a mathematical sense.

c) Place in context the small-world construction in relation to graph and network theory and give examples of its significance and applicability.

Marks: 30

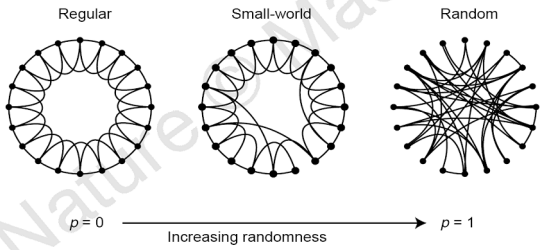

Figure 2:

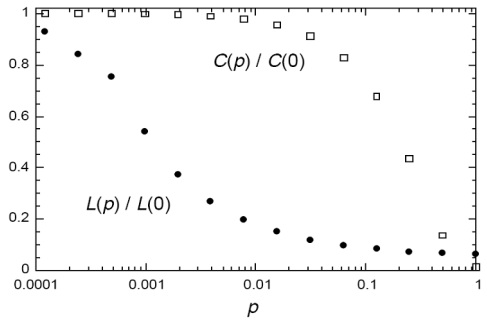

Figure 3:

Question 8

a) Use Figure 4 to explain the experiment described by Weiss, Garfinkel, Spano and Ditto regarding experimental chaos control in cardiac tissue.

b) Use Figure 5 to explain in detail the algorithm used by Ditto et al for chaos control. Figure 4 G-H was obtained in the absence of chaos control. What would be the effect of the algorithm on the observed data?

c) Place in context the chaos control algorithm in relation to linear and nonlinear control and give examples of its significance, applicability and possible problems.

Marks: 30

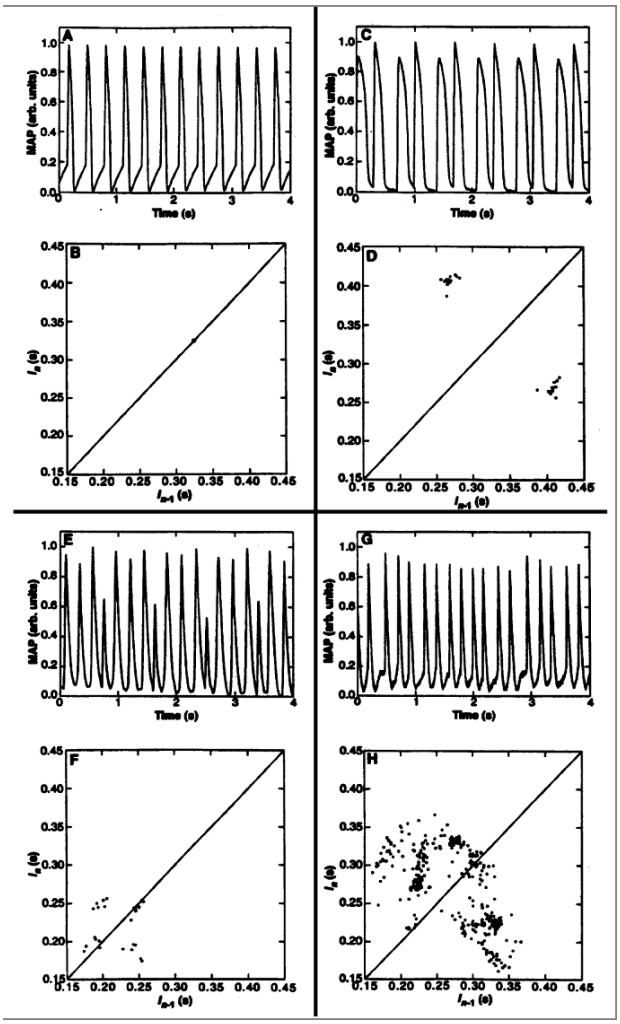

Figure 4:

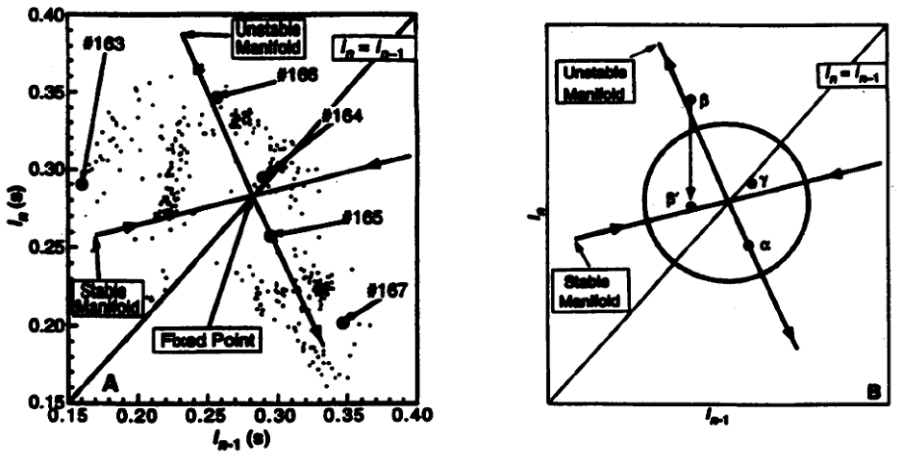

Figure 5: