Part 2: A Little Physical Interpretation

We use the Dimension-less version of the system

$$\begin{cases} (1) \Leftrightarrow \dfrac{dU}{ds} = \dfrac{U}{1+U} - \dfrac{BUV}{B_0 + U} - EU \\ (2) \Leftrightarrow \dfrac{dV}{ds} = \dfrac{CUV}{1+UV} - DV \end{cases}$$

The Dynamic System differs from the previous system by the addition of a term $-EU$, which confers it some very interesting properties.

1) Number of Steady Points

In general the addition of new terms in a Dynamic System results in the creation of new steady points. With previous system we had between 1 and 3 steady points. As we shall see we now have between 3 and 5 points.

2)Behaviour at Infinity

The Dynamic System now has two dissipation terms $-DV$ and $-EU$ (we assume D and E are strictly positive) that help to control the flow along both directions U and V. Better still, their control on the flow is strong enough to prevent the trajectories from escaping to infinity the same way as they were liable to with the previous system (E=0).
Consequently we expect the trajectories to remain bounded (as we shall see a simple Lyapunov function exists to ensure the result).

Proof

With the system $\begin{cases} \dfrac{dU}{ds} = \dfrac{U}{1+U} - \dfrac{BUV}{B_0 + U} \\ \dfrac{dV}{ds} = \dfrac{CUV}{1+UV} - DV \end{cases}$ some combination of parameters would allow some trajectories to escape to infinity as follows $\begin{cases} \dfrac{dU}{ds} = Cte > 0 \\ V \to C/D \end{cases}$

Now with E>0 the coordinate U is ruled by $\dfrac{dU}{ds} = Cte - EU$ so cannot escape to infinity.

3) Influence of E and D

The larger the E and D, the better the flow will be ‘controlled’. For large values of E and D all steady points will be stable – and we will not be able to get any oscillations. As we shall see all but one of the steady points will disappear for large values of D and E. But before disappearing the steady point will go through a phase when they are stable.