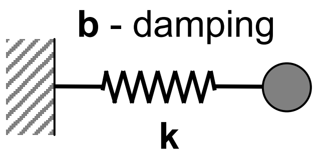
Notes for calculating and measuring thermomechanical noise

20.309

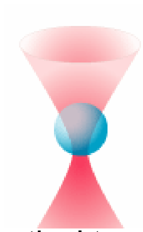
October 22, 2007



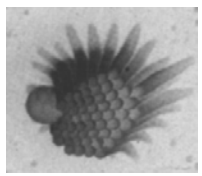
$m\ddot{x} = -kx - b\dot{x} + F_d(t)$ where $F_d(t)$ is driving force

Overdamped Oscillator

$m\ddot{x} << kx$ and bx



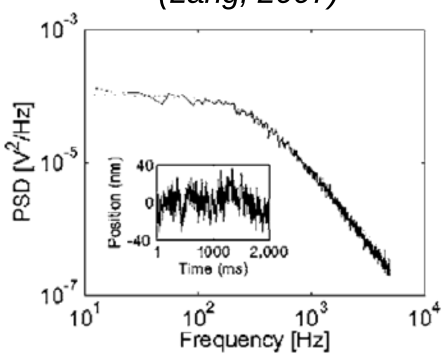
optical trap



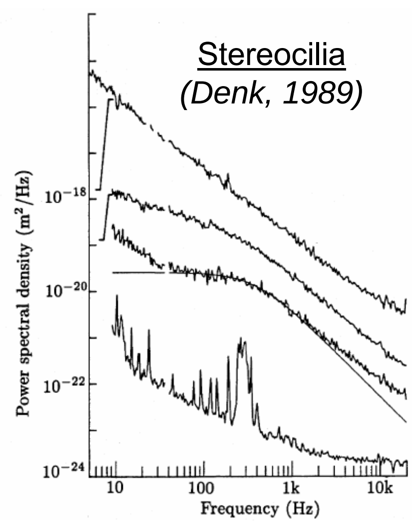
stereocilia

$| H(\omega) | = (1+\omega^2\tau^2)^{-1/2}$ $\tau = b / k$

Optical Trap
(Lang, 2007)

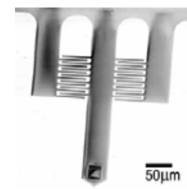


Stereocilia
(Denk, 1989)



Underdamped Oscillator

no approximation



AFM cantilever
(in air or vacuum)

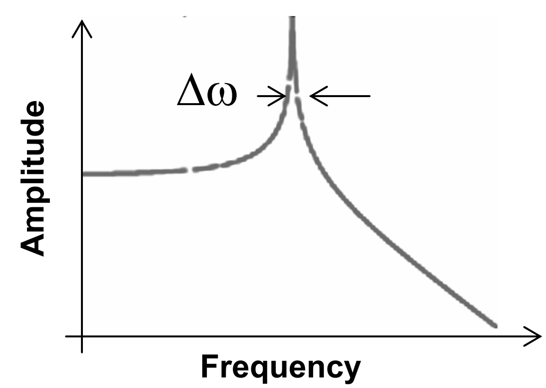
$| H(\omega) | = Q\, m/ k\, [Q^2(1-\omega^2/\omega_o^2)^2 + \omega^2/\omega_o^2]^{-1/2}$

Resonant Frequency

$\omega_o^2 = k / m$

Quality factor

$Q = \omega_o / \Delta\omega$



$Q=\omega_o/2\gamma$

$\gamma=b/2m$

Recall from last time,

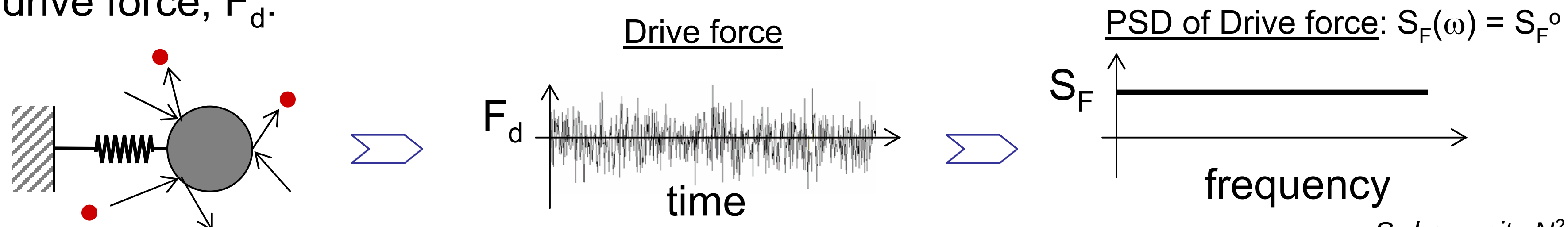
input → output

$$F_d(\omega) / m \longrightarrow \boxed{H(\omega)} \longrightarrow X(\omega)$$

Redefine input (and transfer function),

$$F_d(\omega) \longrightarrow \boxed{H_F(\omega)} \longrightarrow X(\omega) \qquad \text{where } H_F(\omega) = 1/m\ H(\omega)$$

Assume a series of microscopic impulse collisions create white noise fluctuations of drive force, F_d:



S_F has units N^2/Hz

The response in position from driving force is:

$$X(\omega) = F_D(\omega)\ |\ H_F(\omega)\ |$$

If we square both sides, and divide by a long measurement time window, **T**

$$\lim_{T\to\infty} \left\{ (2/T)\ X^2(\omega) = F^2{}_D(\omega)\ |\ H_F(\omega)\ |^2\ \ (2/T) \right\}$$

then we can express this in terms of the PSD,

$$S_x(\omega) = S_F(\omega)\ |\ H_F(\omega)\ |^2 \qquad \textbf{Eq (1)}$$

In lecture, we found that $S_F(\omega) = S_F{}^o = 4\ k_b\ T\ k\ /\ (Q\omega_o)$ by using the Equipartition Theorem

$$\tfrac{1}{2}\ k_b\ T = \tfrac{1}{2}\ k\ \langle x^2 \rangle$$

and by using Parseval's Theorem which gave

$$\langle x^2 \rangle = \int_0^\infty S(f)df$$

Thus, plugging in S_F and H_f into Eq (1) gives

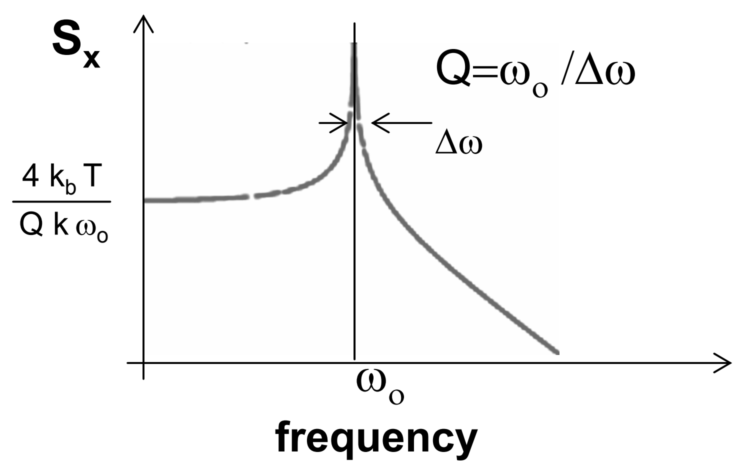
$$S_x(\omega) = \frac{4\ k_b\ T\ Q}{k\ \omega_o}\ \left[\ Q^2(1-\omega^2/\omega_o{}^2)^2 + \omega^2/\omega_o{}^2\ \right]^{-1}$$

The thermal induced position noise, S_x, can be expressed as

Transfer Function, $|G(\omega)|^2$
(p. 19 of Lab Module)

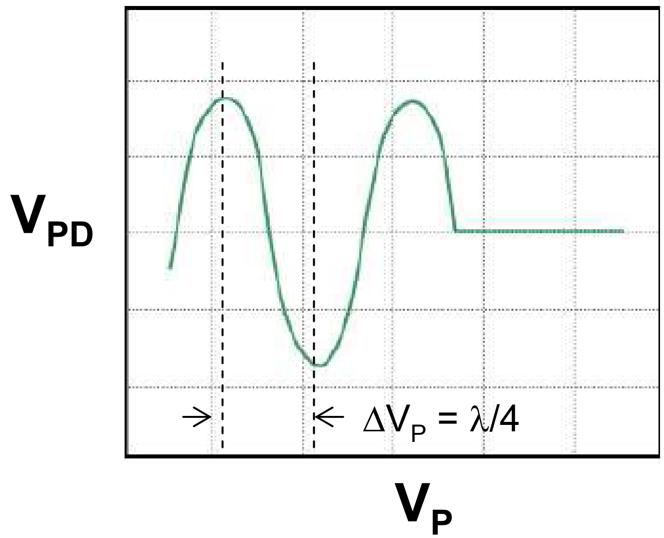
$$S_x(\omega) = \frac{4 k_b T}{Q k \omega_o} \left[(1-\omega^2/\omega_o^2)^2 + \omega^2 / Q^2\omega_o^2 \right]^{-1}$$

Scale factor
(y-intercept of PSD)



Calibrating the PSD measurement, Part I

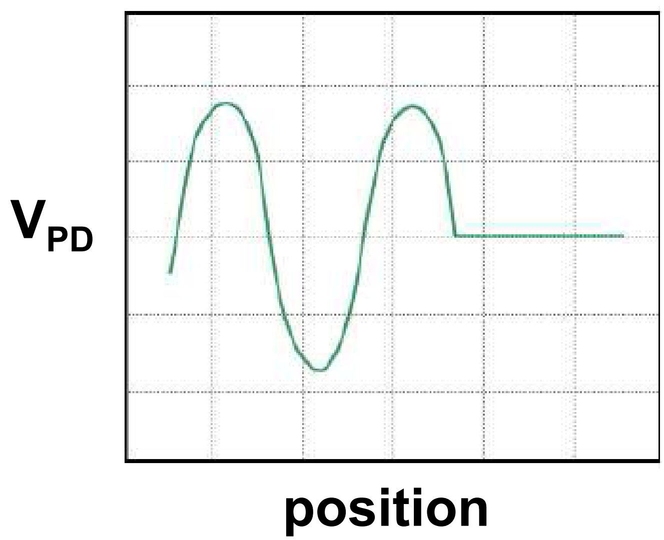
Measure force curve



V_P – voltage to piezo

V_{PD} – voltage from photodiode

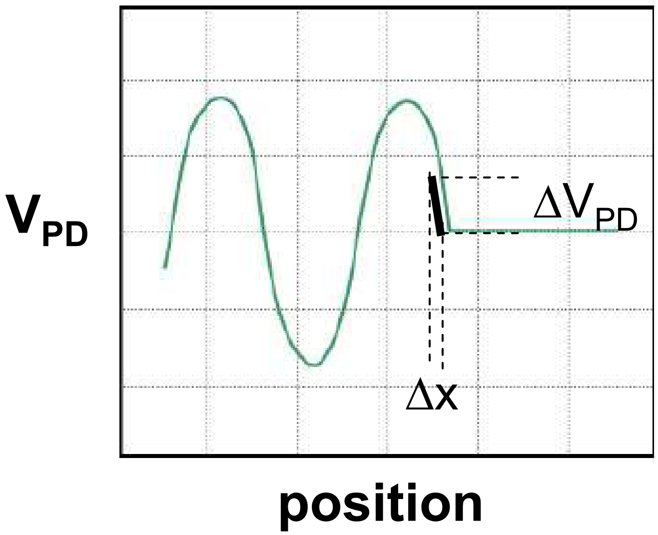
Convert **x-axis** to position by multiplying C_1



$$C_1 = \frac{\lambda/4}{\Delta V_P}$$

Calibrating the PSD measurement, Part II

Determine slope at contact point



$$C_2 = \frac{\Delta x}{\Delta V_{PD}}$$

Multiply PSD measurement by C_2

Note: Labview program acquires PSD in units of $V_{PD}/\sqrt{Hz}$

Minimum detectable force,

$$\langle F^2_{min} \rangle^{1/2} = \sqrt{S_F^o\, B} = \sqrt{4\, k_b\, T\, B\, k\, /\, Q\, \omega_o}$$

where bandwidth, $\mathbf{B} = 1 / \mathbf{T}_{\mathbf{measure}}$

A sensitive force sensor has a low spring constant and high quality factor and resonant frequency