IMPERIAL COLLEGE LONDON
UNIVERSITY OF LONDON

B.Eng., M.Eng and ACGI Examinations 2007

Part 3 and Part 4

Biomedical Engineering

BE3-H36/BE4-H36 Modelling in Biology

Friday, 11th May 2007 10.00-12.00

Marks are shown next to each question.
Please write answers to each Section in separate answer books.

Section A

Give brief answers to **three** of the following four questions.

Total marks for this section: 36

Question 1

a) What is a *fractal* object? Give one mathematical example and one example from the biological world.
b) Statement: *"If the dynamics of a system is chaotic, the attractor of the system is fractal"*. True or false? Explain your answer.

Marks: 12

Question 2

a) Provide a brief definition of *limit cycle*. Illustrate your answer with a sketch of a limit cycle in the phase plane.
b) Consider the system $\ddot{x} + kx = 0$, where k is a positive constant. Can this system have limit cycles? Explain your reasoning.

Marks: 12

Question 3

a) Consider the system

$$\dot{x} = -\frac{r}{2} - \frac{1}{4} + (1+r)x - x^2$$

where $r \in \mathbb{R}$ is a parameter. Find the fixed points x^* of the system and their stability as a function of r. Draw the bifurcation diagram x^* vs r and classify the bifurcations observed.
b) Consider the system

$$\dot{x} = \tanh(x) - rx,$$

where $r \in \mathbb{R}$ is a parameter. Draw the bifurcation diagram x^* vs r and classify the bifurcations observed.

Marks: 12

Question 4

a) What are Monte Carlo methods? Give two of their uses in computational modelling.
b) Consider the following lines of `MATLAB` code:

```
clear; Q=0; Nfinal=5e6;
for N=1:Nfinal;
    pp(1)=rand;
    pp(2)=rand*2;
    pp(3)=rand;
    Q=Q+(pp(1)^2+1/4*pp(2)^2+pp(3)^2<=1)*2;
end
P=Q/Nfinal
```

What is the value of `P` as `Nfinal` goes to infinity? Explain your answer.

Marks: 12

Section B

Answer the following question.

Total marks for this section: 36

Question 5

Consider the following system:

$$\begin{aligned} \dot{x} &= 1-(b+1)x+ax^2y \\ \dot{y} &= bx-ax^2y \end{aligned}$$

where $x, y > 0$ are dimensionless variables and $a, b > 0$ are parameters.

a) Find all the fixed points of the system and classify their linear stability as a function of the parameters a and b.

Marks: 16

b)

i) Find the value b_c of the parameter b at which a Hopf bifurcation occurs as a function of a. Explain your reasoning.

ii) Explain what will happen if we slowly increase b past this value b_c.

Marks: 10

c)

i) Explain briefly the procedure and mathematical results that you would use to prove the existence of a stable limit cycle for this system. (No need to actually prove it; just sketch out the methodology.)

ii) How will the amplitude and shape of the limit cycle change as b becomes larger? Justify your answer.

Marks: 10

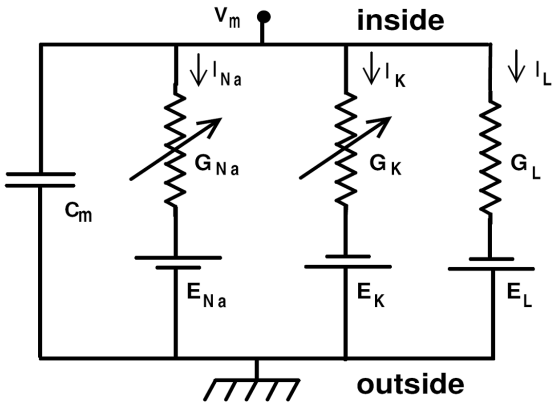
Section C

Answer the following question.

Total marks for this section: 28

Question 6

a) The following figure represents the electrical circuit equivalent proposed by Hodgkin and Huxley in 1952 for a short segment of the squid giant axon. In the figure, V_m, E_{Na}, E_K and E_L are voltages; C_m is a capacitance; and G_{Na}, G_K and G_L are conductances.



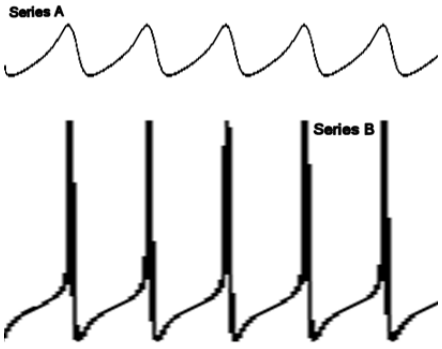
i) Give a biophysical explanation of all the parameters and variables of the figure.

ii) Write down the differential equation for V_m in terms of the parameters and variables of the figure.

iii) In particular, $G_{Na} = f_{Na}(m(V_m), h(V_m))$ and $G_K = f_K(n(V_m))$ correspond to non-constant, voltage-dependent conductances. Give a brief biophysical explanation of the meaning of the variables m, h and n.

Marks: 16

b) Neurons can have a variety of periodic behaviours, including regular spiking. The figure below presents two neuronal time series (Series A and B) of the potential V as a function of time.



i) Sketch the phase portrait $\dot{V}$ vs. V for both series. Discuss the differences.

ii) What will be the observed differences between Series A and B in their power spectra? Illustrate your answer with a sketch.

Marks: 12