Part 4: Behaviour at Infinity of the System

We use the Dimension-less version of the system

$$\begin{cases} (1) \Leftrightarrow \dfrac{dU}{ds} = \dfrac{U}{1+U} - \dfrac{BUV}{B_0 + U} - EU \\ (2) \Leftrightarrow \dfrac{dV}{ds} = \dfrac{CUV}{1+UV} - DV \end{cases}$$

The Dynamic System differs from the previous system by the addition of a term $-EU$, which confers it a crucial property: all trajectories remain bounded

1) A Lyapunov Function

We are going to prove that for (U ,V) far enough from the origin , U+V is a Lyapunov function of the flow. We restrict ourselves to the strictly positive trajectories.

$$\frac{dU}{ds} + \frac{dV}{ds} = \frac{U}{1+U} - \frac{BUV}{B_0 + U} - EU + \frac{CUV}{1+UV} - DV$$

and therefore

$$\frac{dU}{ds} + \frac{dV}{ds} \leq 1 + C - EU - DV \qquad \textbf{QED}$$

2) Behaviour at Infinity

It follows that the trajectories remain bounded even as time goes to infinity. We can therefore apply Poincaré – Bendixson's theorem. Trajectories will therefore converge either to one steady points or to a limit cycle. If we can find a combination of parameters for which all steady points are unstable , then the system will oscillate .