# Biol 398/Math 388 Week 4 Assignment: A second chemostat

The simple model of the chemostat is given by the following equation pair:

$$\frac{dn}{dt} = qu - qn - yV_{\max}\frac{n}{K+n}$$
$$\frac{dy}{dt} = yrV_{\max}\frac{n}{K+n} - qy$$

Here *y* denotes the concentration of yeast, and *n* denotes the concentration of Nitrogen. The states *y* and *n* depend on time, while the parameters $q, r, K, V_{\max}$ are constants. To tie this model to the ter Schure paper, we need to think about a few things.

First, there is a second nutrient that yeast needs, namely Carbon. We will adapt this model to add a carbon nutrient. Next, we will need to figure out values of the parameters. Finally, we need to see what the steady state of the system is. Let's start with a new model:

$$\frac{dn}{dt} = qu_n - qn - yV_{\max}\left(\frac{n}{K_n+n}\right)\left(\frac{c}{K_c+c}\right)$$
$$\frac{dc}{dt} = qu_c - qn - yV_{\max}\left(\frac{n}{K_n+n}\right)\left(\frac{c}{K_c+c}\right)$$
$$\frac{dy}{dt} = ryV_{\max}\left(\frac{n}{K_n+n}\right)\left(\frac{c}{K_c+c}\right) - qy$$

This model is a big adaptation. We have a new equation for *c*, the carbon concentration. The metabolic reaction rate, how the yeast turn carbon and nitrogen into more yeast, is the product of two Michaelis-Menten factors. There are feed rates for each nutrient, $u_n, u_c$, and the Michaelis-Menten factors have different saturation constants, $K_n, K_c$.

Recall that the steady state is the limit of the states as time goes to infinity. Another way of viewing the steady state is that the dynamics become constant in time, so that dn/dt = 0, dc/dt=0, and dy/dt=0.

Complete the following tasks.

(1) Find steady states by replacing the d/dt's with 0. This will involve a bit of algebra.

(2) How do the steady states depend on the feed rates? This will also involve a bit of algebra.

(3) Suppose we replace the product term $\frac{n}{K_n+n}\frac{c}{K_c+c}$ with $\frac{n+c}{K+n+c}$. How might that affect the steady state? Or the dynamics?