Part 1: Dimension-less Modelisation

The full 2D system is
$$\begin{cases} \dfrac{dX}{dt} = \dfrac{aX}{a_0 + X} - \dfrac{bXY}{b_0 + X} - eX \\ \dfrac{dY}{dt} = \dfrac{cXY}{c_0 + XY} - dY \end{cases}$$

We set $U = X/a_0$ which yields
$$\begin{cases} a_0 \dfrac{dU}{dt} = \dfrac{aU}{1+U} - \dfrac{bUY}{B_0 + U} - ea_0 \\ \dfrac{dY}{dt} = \dfrac{cUY}{c_0 / a_0 + UY} - dY \end{cases}$$
with $B_0 = b_0 / a_0$

Now we set $V = Y/\gamma_0$ with $\gamma_0 = c_0 / a_0$ which yields
$$\begin{cases} a_0 \dfrac{dU}{dt} = \dfrac{aU}{1+U} - \dfrac{b\gamma_0 UV}{B_0 + U} - ea_0 U \\ \gamma_0 \dfrac{dV}{dt} = \dfrac{cUV}{1+UV} - d\gamma_0 V \end{cases}$$

Finally we change the time scale $s = at / a_0$ and obtain the dimension-less sytem

$$\begin{cases} (1) \Leftrightarrow \dfrac{dU}{ds} = \dfrac{U}{1+U} - \dfrac{BUV}{B_0 + U} - EU \\ (2) \Leftrightarrow \dfrac{dV}{ds} = \dfrac{CUV}{1+UV} - DV \end{cases}$$

where the new parameters can be expressed as function of the old ones:
$$\begin{cases} B = b\gamma_0 / a = bc_0 / (aa_0) \\ B_o = b_0 / a_0 \\ C = ca_0 / (a\gamma_0) = ca_0{}^2 / (ac_0) \\ D = a_0 d / a \\ E = a_0 e / a \end{cases}$$

The new version of the model has only 5 parameters – against the previous 8. It is also very similar to the dimension-less version of the 2D model we have just studied, which makes comparison of results immediate.

We use the dimension-less version of the Dynamic System in the rest of the study.