TechNote 206

Equations

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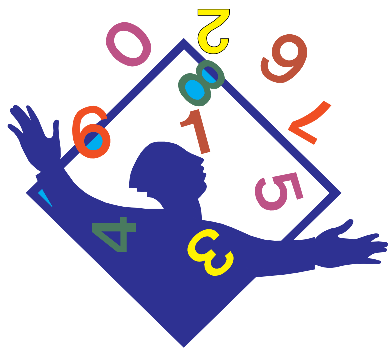
B E A D S ● A B O V E T H E R E S T ™



Contents:

I. Introduction

Values obtained through use of the following equations should be considered estimates, as the equations are based upon a number of theoretical assumptions. Values that are determined empirically or through use of analytical techniques are expected to differ to an extent. Some values *(designated with an *)* are provided on the **Certificates of Analysis** and order paperwork that accompanies microsphere shipments.

II. Equations

A. Solids Content*

$$S = \frac{m \cdot 100}{V_S} \quad \text{OR} \quad m = \frac{S \cdot V_S}{100}$$

where:
- S = weight % solids*
- V_S = volume of suspension (mL)*
- m = mass of microspheres (g)*

(e.g., S = 10 for 5mL suspension, 0.5g microspheres)

B. % Coefficient of Variation (Size Distribution of the Microsphere Population)

$$\%\,CV = \frac{SD}{d} \times 100$$

where:
- % CV = % coefficient of variation (size distribution of the microsphere population)
- SD = standard deviation (µm)* (*Note:* Standard deviation is not provided for all products.)
- d = mean diameter (µm)*

C. # Microspheres/Gram*

$$N = \frac{6 \times 10^{12}}{\pi \cdot \rho_S \cdot d^3}$$

where:
- N = # microspheres/gram for dry powders*
- ρ_S = density of solid sphere (g/cm^3)*
- d = mean diameter (µm)*

D. # Microspheres/mL*

$$N = \frac{6 \times 10^{10} \cdot S \cdot \rho_L}{\pi \cdot \rho_S \cdot d^3}$$

where:
- N = # microspheres/mL for suspensions in water*
- S = weight % solids (for 10% solids suspension, S=10)*
- ρ_L = density of microsphere suspension (g/mL)*
- ρ_L = $100 \cdot \rho_S / [S(1-\rho_S) + (100 \cdot \rho_S)]$
- ρ_S = density of solid sphere (g/cm^3)*
- d = mean diameter (µm)*

E. Surface Area/Gram*

$$A_G = \frac{6 \times 10^{12}}{\rho_S \cdot d}$$

where:
- A_G = surface area/gram for dry powders (µm^2/g)*
- ρ_S = density of solid sphere (g/cm^3)*
- d = mean diameter (µm)*

F. Surface Area/mL*

$$A = \frac{6 \times 10^{10} \cdot S \cdot \rho_L}{\rho_S \cdot d}$$

where:
- A = surface area/mL for suspensions in water (µm^2/g)*
- S = weight % solids (for 10% solids solution, S=10)*
- ρ_L = density of microsphere suspension (g/mL)*
- ρ_L = $100 \cdot \rho_S / [S(1-\rho_S) + (100 \cdot \rho_S)]$
- ρ_S = density of solid sphere (g/cm^3)*
- d = mean diameter (µm)*

G. Parking Area (Surface Charge Density)*

$$P = \frac{1}{1.004 \cdot D_C \cdot \rho_S \cdot d}$$

where:
- P = parking area (Å^2/charge group)*
- D_c = surface charge density or titration value (meq/g)* (*provided in µeq/g on COA*)
- ρ_S = density of solid sphere (g/cm^3)*
- d = mean diameter (µm)*

Notes:
a. Surface titer and parking area are not provided for all products.
b. For a detailed description of parking area, see *"The Particle Doctor®"* section of our March 2001 issue of **Painless Particles®**, which may be downloaded from our website, www.bangslabs.com.

H. # Charge Groups/Microsphere

$$C_M = \frac{\pi \cdot d^2 \cdot 10^8}{P}$$

where:
- C_M = # charge groups/microsphere
- d = mean diameter (µm)*
- P = parking area (Å^2)*

I. Settling Velocity (Specify Medium)

$$v = \frac{1}{18} d^2 (\rho_S - \rho_0) \frac{10^{-6}}{h} G$$

where:
- v = settling velocity (cm/sec)
- d = mean diameter (µm)*
- ρ_S = density of solid sphere (g/cm^3)*
- ρ_0 = medium density (g/cm^3)*
- h = medium viscosity (poise = g/cm/sec)
- G = gravity (980 cm/sec^2)

J. Settling Velocity (In Water)

$$V_m = 5.448 \times 10^{-5} \cdot (\rho_S - 1) \cdot d^2$$

where:
- V_m = maximum settling velocity (cm/sec) for a single microsphere settling in water at room temperature under the influence of normal gravitational force (1G)
- ρ_S = density of solid sphere (g/cm^3)*
- d = mean diameter (µm)*

K. Settling Velocity (In the Centrifuge)

$$V_h^{5\%} = \frac{2}{3} \cdot V_m \cdot G$$

where:
- $V_h^{5\%}$ = true settling velocity or hindered velocity (cm/sec) for a 5% w/w suspension of microspheres settling in water at room temperature
- V_m = maximum settling velocity
- G = multiples of earth gravitation constant, G forces

L. Settling Time

$$t = \frac{h}{V_h^{5\%}} \quad \text{OR} \quad \frac{h}{V_{ch}^{5\%}}$$

where: t = settling time (sec)
h = distance from the top of the liquid to the bottom layer of settled solids (cm)
$V_h^{5\%}$ = true settling velocity or hindered velocity (cm/sec) for a 5% w/w suspension of microspheres settling in water at room temperature under the influence of normal gravitational force (1G)
$V_{ch}^{5\%}$ = hindered settling velocity in the centrifuge (cm/sec) for a 5% w/w suspension of microspheres settling in water at room temperature

M. Surface Saturation (Protein)

$$S = \frac{6}{\rho_S d} \bullet C$$

where: S = amount of representative protein required to achieve surface saturation (mg protein/g microspheres)
ρ_S = density of solid sphere (g/cm^3)*
d = mean diameter (µm)*
C = capacity of microsphere surface for a given protein (mg protein/m^2 of sphere surface)

Notes: a. C ~ 3 mg/m^2 for BSA [MW 65kD],
C ~ 2.5 mg/m^2 for bovine IgG [MW 150kd].[1]
b. See Tech Note 204, *Adsorption to Microspheres*, and TechNote 205, *Covalent Coupling*, for more detailed information.

III. Sample Values

***Table 1:** Sample Values*

Diameter (Microns)	Beads per gram	Beads per mL	Surface Area (µm²/g)	Surface Area (µm²/mL)	Settling Velocity (cm/sec)
0.052	1.3×10^{16}	1.3×10^{15}	1.1×10^{14}	1.1×10^{13}	7.4×10^{-9}
0.100	1.8×10^{15}	1.8×10^{14}	5.7×10^{13}	5.7×10^{12}	2.7×10^{-8}
0.500	1.5×10^{13}	1.5×10^{12}	1.1×10^{13}	1.1×10^{12}	6.8×10^{-7}
1.000	1.8×10^{12}	1.8×10^{11}	5.7×10^{12}	5.7×10^{11}	2.7×10^{-6}
2.500	1.2×10^{11}	1.1×10^{10}	2.3×10^{12}	2.3×10^{11}	1.7×10^{-5}
10.00	1.8×10^{9}	1.8×10^{8}	5.7×10^{11}	5.7×10^{10}	2.7×10^{-4}
25.00	1.2×10^{8}	1.2×10^{7}	2.3×10^{11}	2.3×10^{10}	2.0×10^{-3}
108.0	1.4×10^{6}	---	5.3×10^{10}	---	---
500.0	1.4×10^{4}	---	1.1×10^{10}	---	---

Notes: a. Calculations for 0.052-25.0µm are based on a suspension of polystyrene microspheres (density = 1.05 g/cm^3) at 10% solids (w/v).
b. 108 and 500µm diameter microsphere calculations compositions are based on compositions of poly(styrene/2% divinylbenzene), density = 1.06 g/cm^3, and the calculations are based on dry presentation.

IV. References

1. **Cantarero, L.A., J.E. Butler, J.W. Osborne.** 1980. The adsorptive characteristics of protein for polystyrene and their significance for solid-phase immunoassays. *Analytical Biochemistry,* 105: 375-382.
2. **Bangs, L.B.** 1987. Uniform latex particles. Indianapolis: Seragen Diagnostics, Inc.